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ABSTRACT

This workbook is designed to be used as a review at the end of a college level course in algebra and trigonometry. Four review sheets containing 117 problems provide hints and detailed solutions for each problem. (YP)

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ALGEBRA AND TRIGONOMETRY

REVIEW WORKBOOK

Exercises with hints and complete solutions in algebra and trigonometry.

by

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February 1989

SE 050 443

This workbook was designed to be used as a review at the end of a college level course in Algebra and Trigonometry. The student should first attempt to complete each review sheet on his/her own. Only if necessary should the hint column be consulted. He may then check his answers by reading through the detailed solution.

I would like to thank my colleague , Professor Marcia Weisser, for reading through this manuscript, and for her helpful suggestions.

Dr. Sharon Goodman-Petrushka

February 1989

REVIEW SHEET # 1

1. Solve the following quadratic equation by completing the square:

$$x^2 + 8x - 33 = 0 \ .$$

2. Compute the discriminant of the quadratic equation $x^2 + 2x + 9 = 0$ and discuss the nature of the roots.

3. Solve : $\sqrt{3x - 5} - 2 = 0$.

4. Solve: $t^{\frac{2}{3}} - 2t^{\frac{1}{3}} - 24 = 0$.

5. Let $f(x) = \sqrt{2x + 8}$
   a. Find the domain of f .
   b. Find the range of f .
   c. Find $f(-2) + f(2)$.

6. Let $f(x) = 3x^2 + 5$ and $g(x) = \frac{6}{x} - 2$.

   Find $(f \circ g)(x)$ and $(g \circ f)(x)$.

7. Solve: $|7 - 2x| = 5$.

8. Solve: $x^2 + x + 1 > 0$.

9. Solve: $\frac{(x + 2)(2x - 1)}{(x + 3)} \leq 0$

10. Solve using Cramer's Rule (if possible):

$$\begin{aligned} 2x + y &= 1 \\ x - 3y &= 11 \end{aligned}$$

11. Solve without using Cramer's Rule:

$$\begin{aligned} 2x + 3y + z &= 1 \\ x + y - 2z &= -4 \\ 3x + 2y + 5z &= 11 \end{aligned}$$

12. An estate of \$200,000 was divided among two daughters and one son. The daughters received equal shares and their portion (together) totalled \$100,000 more than the son's share. Find what each son and daughter received.

13. Write a logarithmic expression equivalent to $y = b^5$.

14. Solve : $\log_a 1 = x$.

15. If $\log_b 5 = r$ and $\log_b 3 = s$, express each of the following in terms of r and s:
    a. $\log_b(15)$ c. $\log_b(\sqrt{5})$
    b. $\log_b(.6)$ d. $\log_b(75)$.

16. Solve for x : $\log_3(x^2 + 2x) = 2$.

17. Express the following angles in radians:
    a. 90° b. 45° c. 360° d. 150° e. 100°

18. Sketch the graph of the function $y = \cos x$. What is its period?

19. Evaluate each of the following:
    a. $\cos \frac{\pi}{4}$ b. $\tan 135^\circ$ c. $\cos \frac{5\pi}{4}$ d. $\sec \frac{7\pi}{4}$ e. $\sin(-45^\circ)$

20. Evaluate each of the following:
    a. $\sin \frac{\pi}{2}$ b. $\cos(-180^\circ)$ c. $\cot 2\pi$ d. $\sin(-270^\circ)$

21. Consider $y = 5 \sin (3x - \pi)$
    Find the amplitude, period and phase shift. Then sketch one cycle of the graph.

22. Given that $\sin \theta = \frac{3}{8}$ where $\frac{\pi}{2} < \theta < \pi$. Find:
    a. $\cos \theta$ b. $\tan \theta$ c. $\sin 2\theta$ d. $\cos \frac{\theta}{2}$ e. $\cos(\theta + \frac{\pi}{2})$

23. a. Prove the identity $\frac{\tan x - \csc x}{\sec x} = \sin x - \cot x$

    b. Simplify $2 \cos^2 \frac{1}{2}x - \cos x$

24. Express in the form a + b i :
    a. $(5 + 3i) - (6 + 2i)$ b. $(3 - 2i)(4 + 7i)$

25. Assume that x and y are real numbers and solve for x and y :

$$2x + y - i = 4 + (x - y)\,i$$

26. Use the binomial theorem to expand $(x - 2y^2)^4$.

27. Consider $p(x) = x^5 + 2x^4 + 7x^3 - 4x^2 - 26x + 20$.

a. What is the remainder, r(x), when p(x) is divided by x - 3?
b. Using synthetic division, find the remainder , r(x), when p(x) is divided by x + 2 .
c. Factor p(x) completely.
d. Determine all of the zeros (roots) of p(x) , with multiplicities.

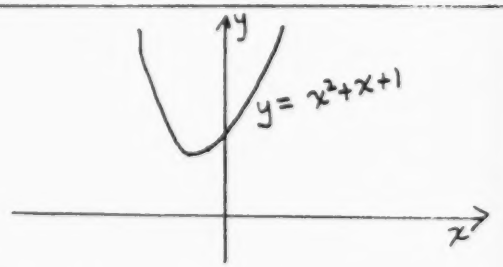
28. Find the length of an arc of a circle given a central angle of 135° and a radius of 6 inches.

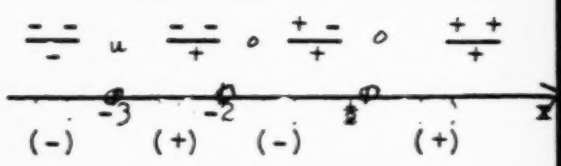
29. A polynomial of degree 7 with rational coefficients has

2 , i , $\sqrt{5}$ and $1 - 2i$ as roots. Find the other roots.

REVIEW SHEET # 1 - SOLUTIONS

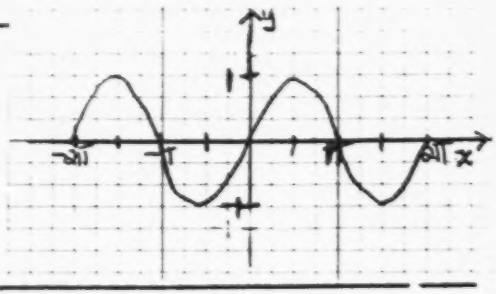
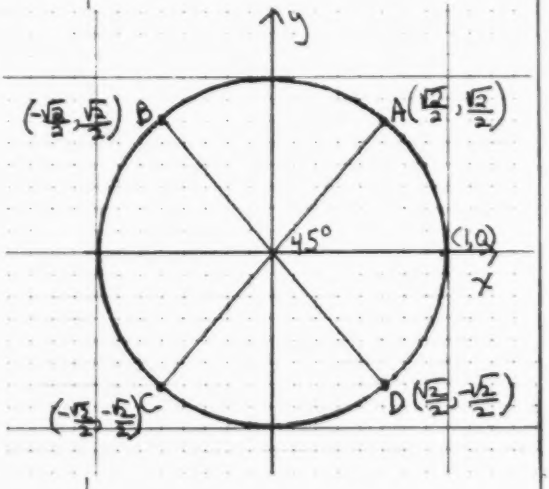
Problem	Hint	Solution
1. Solve by completing the square: $x^2 + 8x - 33 = 0$	$x^2 + 8x + __ = 33 + __$ $(x + __)^2 = 33 + __$	We add to both sides: $(\frac{1}{2}\cdot 8)^2 = 4^2 = 16$ $x^2 + 8x + 16 = 33 + 16$ $(x + 4)^2 = 49$ $x + 4 = \pm\sqrt{49} = \pm 7$ $x + 4 = 7$ or $x + 4 = -7$ $x = 3$ or $x = -11$
2. Compute the discriminant of the equation $x^2 + 2x + 9 = 0$ and discuss the nature of the roots.	The discriminant : $b^2 - 4ac = ?$	$a = 1$, $b = 2$, $c = 9$ so $b^2 - 4ac = 2^2 - 4(1)(9) = -32$. Since the <u>discriminant</u> is <u>negative</u> , the roots are <u>complex conjugate numbers.</u>
3. Solve $\sqrt{3x - 5} - 2 = 0$	First set $\sqrt{3x - 5} = 2$, then square both sides.	$\sqrt{3x - 5} = 2$ $3x - 5 = 2^2 = 4$ $3x = 9$ so that $x = 3$. Since there are radicals in the equation, we must check our answer: $\sqrt{3(3) - 5} - 2 = \sqrt{4} - 2 = 0$
4. Solve $t^{\frac{2}{3}} - 2t^{\frac{1}{3}} - 24 = 0$	Let $u = t^{\frac{1}{3}}$	Letting $u = t^{\frac{1}{3}}$, so $u^2 = t^{\frac{2}{3}}$ $u^2 - 2u - 24 = 0$ $(u - 6)(u+4) = 0$ $u = 6$ or $u = -4$ $t^{\frac{1}{3}} = 6$ or $t^{\frac{1}{3}} = -4$ $t = 6^3 = 216$ or $t = (-4)^3 = -64$ so $t = 216$, -64

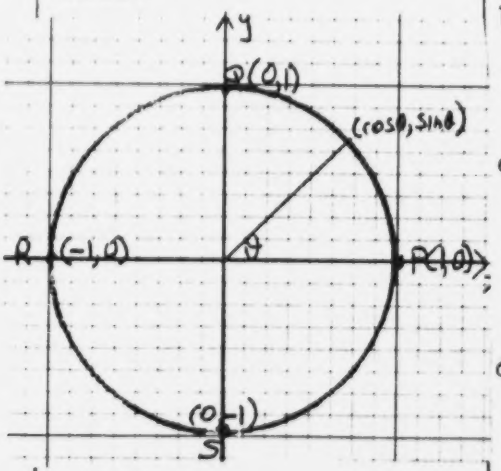
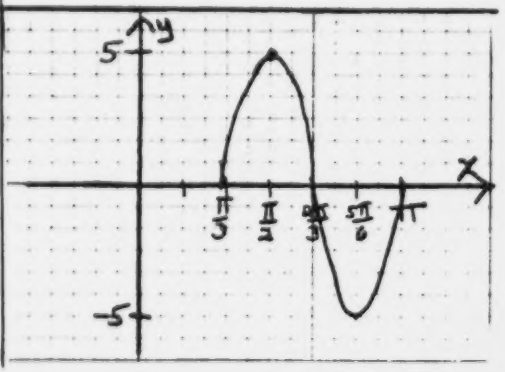
Problem	Hint	Solution
5. Let $f(x) = \sqrt{2x + 8}$ a) Find the domain of f. b) Find the range of f. c) Find $f(-2) + f(2)$.	$\sqrt{u}$ is defined for $u \geq 0$	a) We must have $2x + 8 \geq 0$, or $2x \geq -8$ or $x \geq -4$. b) The range is all real numbers greater than or equal to zero. c) $f(-2) = \sqrt{2(-2) + 8} = \sqrt{4} = 2$ $f(2) = \sqrt{2(2) + 8} = \sqrt{12} = 2\sqrt{3}$ so $f(-2) + f(2) = 2 + 2\sqrt{3}$.
6. Let $f(x) = 3x^2 + 5$ and $g(x) = \frac{6}{x} - 2$ find $(f \circ g)(x)$ and $(g \circ f)(x)$	$(f \circ g)(x) = f(g(x))$ $(g \circ f)(x) = g(f(x))$	$(f \circ g)(x) = f(g(x)) =$ $f\left(\frac{6}{x} - 2\right) =$ $3\left(\frac{6}{x} - 2\right)^2 + 5$. $(g \circ f)(x) = g(f(x)) =$ $g(3x^2 + 5) =$ $\frac{6}{3x^2 + 5} - 2$.
7. Solve $\lvert 7 - 2x \rvert = 5$	$\lvert x \rvert = a$ is equivalent to $x = a$ or $x = -a$ if a is a positive real number.	$7 - 2x = 5$ or $7 - 2x = -5$ $-2x = -2$ or $-2x = -12$ $x = 1$ or $x = 6$
8. Solve $x^2 + x + 1 > 0$	Sketch the graph of the parabola $y = x^2 + x + 1$. Note that there are no real roots of the equation $x^2 + x + 1 = 0$.	 From the graph, we see that $x^2 + x + 1 > 0$ for all real numbers. Thus the answer is <u>all real numbers</u>.

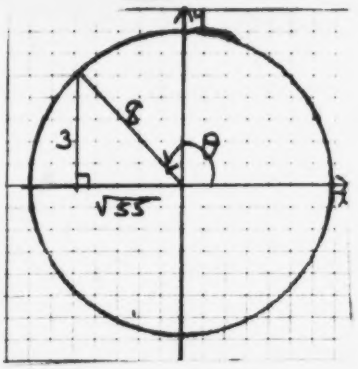
Problem	Hint	Solution
9. Solve $\frac{(x+2)(2x-1)}{(x+3)} < 0$	Use the number line where -3, -2, $\frac{1}{2}$ are the critical values.	 In each interval, we test a number in the expression in left side of the inequality and mark a + or - sign on the number line above. Our solution set consists of those intervals marked with a - sign : $x < -3$ or $-2 < x < \frac{1}{2}$ or, in interval notation: $(-\infty, -3) \cup (-2, \frac{1}{2})$
10. Solve using Cramer's Rule: $2x + y = 1$ $x - 3y = 11$	Use determinants : $x = D_x/D$ $y = D_y/D$	$x = \frac{D_x}{D} = \frac{\begin{vmatrix} 1 & 1 \\ 11 & -3 \end{vmatrix}}{\begin{vmatrix} 2 & 1 \\ 1 & -3 \end{vmatrix}} = \frac{-3-11}{-6-1}$ $= \frac{-14}{-7} = 2$ $y = \frac{D_y}{D} = \frac{\begin{vmatrix} 2 & 1 \\ 1 & 11 \end{vmatrix}}{\begin{vmatrix} 2 & 1 \\ 1 & -3 \end{vmatrix}} = \frac{22-1}{-6-1}$ $= \frac{21}{-7} = -3$. Thus, $x = 2$, $y = -3$
11. Solve without using Cramer's Rule: $2x + 3y + z = 1$ $x + y - 2z = -4$ $3x + 2y + 5z = 11$	Try to get the system into the form $x + ay + bz = c$ $y + dz = e$ $z = f$	Interchange the first two equations: $x + y - 2z = -4$ $2x + 3y + z = 1$ $3x + 2y + 5z = 11$ (continued)

Problem	Hint	Solution
11. (continued)		Replace second equation by itself minus twice the first. Replace third equation by itself minus three times the first: $x + y - 2z = -4$ $y + 5z = 9$ $-y + 11z = 23$ Replace third equation by itself plus second: $x + y - 2z = -4$ $y + 5z = 9$ $16z = 23$ Working from the bottom up: $z = \frac{32}{16} = 2$, $y + 5(2) = 9$, or $y = 9 - 10 = -1$ $x + (-1) - 2(2) = -4$, or $x = -4 + 1 + 4 = 1$, so $x = 1$, $y = -1$, $z = 2$
12. An estate of $200,000 was divided among two daughters and one son. The daughters received equal shares, and their portions (together) totalled $100,000 more than the son's share. Find what each son and daughter received.	Let d = each daughter's share in dollars. Let s = son's share in dollars. Find two equations in the two variables d and s .	$2d + s = 200,000$ $2d = s + 100,000$ or $2d + s = 200,000$ add $2d - s = 100,000$ $4d = 300,000$ $d = \frac{300,000}{4} = 75,000$ $s = 200,000 - 2(75,000) =$ $50,000$ Each daughter got $75,000, and the son got $50,000.

Problem	Hint	Solution
13. Write a logarithmic expression equivalent to $y = b^5$	The logarithm is the exponent.	$\log_b y = 5$
14. Solve: $\log_a 1 = x$	Change to exponential form.	$a^x = 1$ $x = 0$
15. If $\log_b 5 = r$ and $\log_b 3 = s$, Express each of the following in terms of r and s . a) $\log_b(15)$ b) $\log_b(.6)$ c) $\log_b(\sqrt{5})$ d) $\log_b(75)$	$\log_b(uv) = \log_b u + \log_b v$ $\log_b(\frac{u}{v}) = \log_b u - \log_b v$ $\log_b(u^r) = r \log_b u$ r a rational number.	a) $\log_b(15) = \log_b(3 \cdot 5) =$ $\log_b 3 + \log_b 5 = s + r$ b) $\log_b(.6) = \log_b(\frac{6}{10}) =$ $\log_b(\frac{3}{5}) =$ $\log_b 3 - \log_b 5 = s - r$ c) $\log_b(\sqrt{5}) = \log_b(5^{\frac{1}{2}}) =$ $\frac{1}{2}\log_b 5 = \frac{1}{2}r$ d) $\log_b(75) = \log_b(3 \cdot 5^2) =$ $\log_b 3 + \log_b(5^2) =$ $\log_b 3 + 2 \log_b 5 = s + 2r$
16. Solve for x : $\log_3(x^2 + 2x) = 2$	Change to exponential form.	$3^2 = x^2 + 2x$ $9 = x^2 + 2x$ $x^2 + 2x - 9 = 0$ By the quadratic formula: $x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-9)}}{2(1)}$ $= \frac{-2 \pm \sqrt{40}}{2} = \frac{-2 \pm 2\sqrt{10}}{2}$ $= -1 \pm \sqrt{10}$

Problem	Hint	Solution
17. Express the following angles in radians: a) 90° b) 45° c) 360° d) 150° e) 100°	To change from degrees to radians, multiply the number of degrees by $\frac{\pi}{180^\circ}$.	a) $90^\circ = 90^\circ \cdot \frac{\pi}{180^\circ} = \frac{\pi}{2}$ b) $45^\circ = 45^\circ \cdot \frac{\pi}{180^\circ} = \frac{\pi}{4}$ c) $360^\circ = 360^\circ \cdot \frac{\pi}{180^\circ} = 2\pi$ d) $150^\circ = 150^\circ \cdot \frac{\pi}{180^\circ} = \frac{5\pi}{6}$ e) $100^\circ = 100^\circ \cdot \frac{\pi}{180^\circ} = \frac{5\pi}{9}$
18. Sketch the graph of the function $y = \cos x$. What is its period:	$\cos(x + ?) = \cos x$	
19. Evaluate each of the following: a) $\cos \frac{\pi}{4}$ b) $\tan 135^\circ$ c) $\cos \frac{5\pi}{4}$ d) $\sec \frac{7\pi}{4}$ e) $\sin(-45^\circ)$	All of the given angles have a reference angle of 45°. We sketch the unit circle and the terminal points of these angles. 	a) The terminal side of $\frac{\pi}{4} = 45^\circ$ intersects the unit circle at the point A. Thus the cosine of this angle is the x coordinate of the point A which is $\frac{\sqrt{2}}{2}$. b) The terminal side of 135° intersects the unit circle at point B. Thus $\sin 135^\circ = \frac{\sqrt{2}}{2}$ and $\cos 135^\circ = \frac{-\sqrt{2}}{2}$. So, $\tan 135^\circ = \frac{\sin 135^\circ}{\cos 135^\circ} = \frac{\frac{\sqrt{2}}{2}}{\frac{-\sqrt{2}}{2}} = -1$

Problem	Hint	Solution
19: (continued)		c) $\frac{5\pi}{4}$ intersects the unit circle at point C . Thus cos $\frac{5\pi}{4}$ is equal to the x coordinate of this point which is $-\frac{\sqrt{2}}{2}$. d) $\frac{7\pi}{4}$ intersects the unit circle at point D. Thus, sin $\frac{7\pi}{4}$ is equal to the y coordinate of this point which is $-\frac{\sqrt{2}}{2}$. e) -45° intersects the unit circle at point D. Thus $\sin(-45^\circ) = -\frac{\sqrt{2}}{2}$.
20. Evaluate each of the following: a) sin $\frac{\pi}{2}$ b) cos(-180°) c) cot 2π d) sin(-270°)	All of these angles are quadrantal angles i. e. their terminal sides lie along the x or y axis.  	a) $\frac{\pi}{2}$ intersects the unit circle at point Q . Thus, $\sin\frac{\pi}{2} = 1$. b) -180° intersects the unit circle at point R. Thus $\cos(-180^\circ) = -1$. c) 2π intersects the unit circle at point P . Thus cot $2\pi = \frac{\cos 2\pi}{\sin 2\pi}$ is undefined, since $\sin 2\pi = 0$. d) -270° intersects the unit circle at the point Q . Thus, $\sin(-270^\circ) = 1$
21. Let $y = 5\sin(3x - \pi)$ Find the amplitude, period, phase shift. Sketch one cycle of the graph	$y = 5\sin 3(x - \frac{\pi}{3})$ amplitude = 5 period = $\frac{2\pi}{3}$ phase shift: $\frac{\pi}{3}$ (to right)	

Problem	Hint	Solution
22. Given that $\sin\theta = \frac{3}{8}$, where $\frac{\pi}{2} < \theta < \pi$. Find a) $\cos\theta$ b) $\tan\theta$ c) $\sin 2\theta$ d) $\cos\frac{1}{2}\theta$ e) $\cos(\theta + \frac{\pi}{2})$	 $\sin\theta = \frac{3}{8} = \frac{y}{r}$ so $y = 3$, $r = 8$. $x^2 + y^2 = r^2$ $x^2 + 9 = 64$ $x^2 = 55$, $x = \sqrt{55}$ note: since $\frac{\pi}{2} < \theta < \pi$, $\pi < 2\theta < 2\pi$ so that 2θ is in quadrant III or quadrant IV, and $\frac{\pi}{4} < \frac{\theta}{2} < \frac{\pi}{2}$ so that $\frac{\theta}{2}$ is in quadrant I	a) $\cos\theta = \frac{x}{r} = -\frac{\sqrt{55}}{8}$ b) $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{3}{8}}{-\frac{\sqrt{55}}{8}} = \frac{-3}{\sqrt{55}}$ c) $\sin 2\theta = 2\sin\theta\cos\theta = 2 \cdot \frac{3}{8} \cdot -\frac{\sqrt{55}}{8} = -\frac{3\sqrt{55}}{32}$ d) $\cos^2(\frac{1}{2}\theta) = \frac{1}{2}(1 + \cos\theta) = \frac{1}{2}(1 - \frac{\sqrt{55}}{8})$ $\cos\frac{1}{2}\theta = \sqrt{\frac{1}{2}(1 - \frac{\sqrt{55}}{8})} = \sqrt{\frac{8 - \sqrt{55}}{4}}$ note that $\cos\frac{1}{2}\theta$ is positive since $\frac{1}{2}\theta$ is in quadrant I. e) $\cos(\theta + \frac{\pi}{2}) = \cos\theta\cos\frac{\pi}{2} - \sin\theta\sin\frac{\pi}{2}$ $= \cos\theta \cdot 0 - \sin\theta \cdot 1 =$ $-\sin\theta = \frac{-3}{8}$.
23. a) Prove $\frac{\tan x - \csc x}{\sec x}$ $= \sin x - \cot x$	Rewrite the left side as two fractions and change every term to sine and cosine.	$\frac{\tan x - \csc x}{\sec x} =$ $\frac{\tan x}{\sec x} - \frac{\csc x}{\sec x} =$ $\frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}} - \frac{\frac{1}{\sin x}}{\frac{1}{\cos x}} =$ $\sin x - \frac{\cos x}{\sin x} =$ $\sin x - \cot x$

Problem	Hint	Solution
23. b) Simplify $2\cos^2 \frac{1}{2}x - \cos x$	Half Angle Formula	$\cos^2 \frac{1}{2}x = \frac{1}{2}(1 + \cos x)$ $2\cos^2 \frac{1}{2}x = 1 + \cos x$ $2\cos^2 \frac{1}{2}x - \cos x =$ $1 + \cos x - \cos x = 1$
24. Express in the form $a + bi$ a) $(5+3i) - (6+2i)$ b) $(3-2i)(4+7i)$	$i^2 = -1$	a) $(5 + 3i) - (6 + 2i) =$ $(5 - 6) + (3i - 2i) = -1 + i$ b) $(3 - 2i)(4 + 7i) =$ $12 + 21i - 8i - 14i^2 =$ $12 + 13i - 14(-1) =$ $26 + 13i$
25. Assuming x and y to be real numbers, solve for x and y $2x + y - i =$ $4 + (x - y)i$	$a + bi = c + di$ if and only if $a = c$ and $b = d$	$2x + y = 4$ $x - y = -1$ Adding the equations: $3x = 3$ $x = 1$ and $y = 2$
26. Use the binomial theorem to expand $(x - 2y^2)^4$	$(a + b)^n =$ $a^n + \binom{n}{1} a^{n-1}b^1 +$ $\binom{n}{2} a^{n-2}b^2 + \ldots + b^n$	$(x - 2y^2)^4 =$ $x^4 + \binom{4}{1}x^3(-2y^2)^2 + \binom{4}{2}x^2(-2y^2)$ $+ \binom{4}{3}x^1(-2y^2)^3 + (-2y^2)^4 =$ $x^4 + 4x^3(-2)y^2 + 6x^2(-2)^2y^4 +$ $4x(-2)^3y^6 + (-2)^4y^8 =$ $x^4 - 8x^3y^2 + 24x^2y^4 - 32xy^6$ $+16y^8$

Problem

27. Let $p(x) = x^5+2x^4+7x^3-4x^2-26x+20$

a) Find the remainder r(x) when p(x) is divided by $x - 3$.

b) Find the remainder r(x) when p(x) is divided by $x+2$

c) Factor p(x) completely.

d) Determine all the zeros of p(x)

Hint

If a polynomial p(x) has a rational root, c/d , then c divides the constant term of p(x) and d divides the leading coefficient.

Use synthetic division to test the possible rational roots.

Solution

a) To divide p(x) by x-3 we use synthetic division:

3	1	2	7	-4	-26	20
		3	15	66	186	480
	1	5	22	62	160	500

The remainder is 500 which is also equal to p(3).

b)

-2	1	2	7	-4	-26	20
		-2	0	-14	36	-20
	1	0	7	-18	10	0

The remainder is 0 when p(x) is divided by x+2.

Thus p(x) has x+2 as a factor,

and

$p(x) = (x+2)(x^4+ 7x^2-18x+10)$

c) To factor completely, we must factor the polynomial

$x^4 + 7x^2 - 18x + 10.$

Its possible rational roots are ± 1 , ± 2, ± 5 , ± 10.

Using synthetic division, we test the possibilities

1	1	0	7	-18	10
		1	1	8	-10
	1	1	8	-10	0

Thus x-1 is a factor of the polynomial and

$p(x)=(x+2)(x-1)(x^3+x^2+8x-10)$.

The possible rational roots of $x^3+x^2+8x-10$ are ± 1 , ± 2, ± 5, ± 10.

1	1	1	8	-10
		1	2	10
	1	2	10	0

Thus x-1 is a factor, and

Problem	Hint	Solution
26. (continued)		$p(x) = (x+2)(x-1)^2(x^2+2x+10)$ d) The zeros of p(x) are then $x = -2 , 1 , 1 , -1+3i, -1-3i.$ (The last two are obtained by using the quadratic formula on $x^2 + 2x + 10 = 0$.)
28. Find the length of an arc of a circle, given a central angle of 135° and a radius of 6 inches.	$s = \theta r$ where θ is the central angle, in radians.	135° = ? radians $\theta = 135^\circ = 135^\circ \cdot \frac{\pi}{180^\circ} = \frac{3\pi}{4}$ $s = \frac{3\pi}{4} \cdot 6 = \frac{9\pi}{2} = 14.13$ inches.
29. A polynomial of degree 7 with rational coefficients has 2 , i , $\sqrt{5}$, and 1-2i as roots. Find the other roots.	In order to have rational coefficients, irrational and complex roots must appear in conjugate pairs.	The roots are $2 , i , -i , \sqrt{5} , -\sqrt{5} ,$ $1 - 2i , 1 + 2i .$

REVIEW SHEET # 2

1. Solve the quadratic equation $x^2 - 7x + 4 = 0$ by completing the square.

2. Compute the discriminant of the quadratic equation $4x^2 - 20x + 25 = 0$ and discuss the nature of the roots.

3. Solve: $\sqrt{7 + 4x} + 9 = 0$.

4. Solve: $x^4 - 3x^2 + 2 = 0$.

5. Let $g(x) = 5 - 2x^2$
   a) Find the domain of f .
   b) Find the range of f .
   c) Sketch the graph of f .
   d) Find and simplify $\frac{f(a+h) - f(a)}{h}$, for $h \neq 0$.

6. Let $f(x) = \sqrt{2x + 3}$ and $g(x) = x^3 + 1$. Find $(f \circ g)(x)$ and $(g \circ f)(x)$.

7. Solve $|2x - 1| < 8$.

8. Solve: $-x^2 + 2x - 9 > 0$.

9. Solve: $\frac{x - 1}{x + 3} > 5$.

10. Solve <u>without</u> using Cramer's Rule:

$$2x + 5y + z = -2$$
$$-x + y + 2z = 5$$
$$3x + y + z = 2 \quad .$$

11. Solve, if possible, using Cramer's Rule:

$$x + 2y = 5$$
$$3x + 6y = 1 \quad .$$

12. Mixture A is 14% alchohol and mixture B is 8% alchohol. How many liters of each should be mixed to get ten liters of a mixture that is 12% alchohol?

13. Write $z = r^4$ as an equivalent logarithmic expression.

14. Solve for x : a) $\log_x 16 = 2$ b) $\log_9 x = -\frac{1}{2}$.

15. Express as a single logarithm:

$$5 \log_b x + \frac{\log_b y}{7} - \log_b 6 \ .$$

16. Sketch the graph of the function $y = \log_5 x$.

17. Express the following angles in degrees:

a) $\frac{5\pi}{3}$ b) $\frac{7\pi}{6}$ c) 3π d) $\frac{3\pi}{4}$

18. Sketch the graph of the function $y = \sin x$. What is its period?

19. Evaluate each of the following:

a) $\cos(-\frac{\pi}{2})$ b) $\tan(180^\circ)$ c) $\sec 2\pi$ d) $\sin 270^\circ$

20. Evaluate each of the following:

a) $\sin \frac{\pi}{6}$ b) $\cos 150^\circ$ c) $\tan \frac{7\pi}{6}$ d) $\csc 330^\circ$ e) $\cot(-\frac{5\pi}{6})$.

21. Given that $\cos \theta = \frac{2}{7}$ where $\frac{3\pi}{2} < \theta < 2\pi$. Find:

a) $\sin \theta$ b) $\sin 2\theta$ c) $\cot \theta$ d) $\cos \frac{1}{2}\theta$ e) $\sin(\theta + \frac{\pi}{2})$.

22. Consider $y = -2 \cos(\frac{x}{4} + \frac{\pi}{2})$.

Find the amplitude, period and phase shift. Then sketch one cycle of the graph.

23. Prove: a) $\frac{1}{1+\cos x} - \frac{1}{1 - \cos x} = -2 \cot x \csc x$

b) $\cos^4 y - \sin^4 y = \cos 2y$.

24. Express $\frac{5+i}{6-2i}$ in the form $a + bi$.

25. Assume that x and y are real numbers and solve for x and y :

$$3x - y + 5yi = 7 + 3i .$$

26. Let $p(x) = x^4 - x^3 + 5x^2 - 7x - 14$.

a) List all of the possible rational roots of $p(x)$.
b) Factor $p(x)$ completely .
c) List all of the roots of $p(x)$.

27. Find the zeros , with multiplicity, of the polynomial

$$p(x) = 3x\,(x+2)^3\,(2x-5) .$$

28. Use the binomial theorem to expand $(x + \frac{2}{y})^5$.

29. An arc 6 ft long on a circle of radius 24 ft , subtends an angle of how many radians? How many degrees are in this angle to the nearest degree?

REVIEW SHEET # 2 - SOLUTIONS

1. Solve by completing the square:

$x^2 - 7x + 4 = 0$

$x^2 - 7x + __ = -4 + __$

$(x + __)^2 = -4 + __$

We add $(-7/2)^2 = (49/4)$ to both sides. Thus

$x^2 - 7x + \frac{49}{4} = -4 + \frac{49}{4}$

$(x - \frac{7}{2})^2 = \frac{33}{4}$

$x - \frac{7}{2} = \pm \frac{33}{4} = \pm \frac{\sqrt{33}}{2}$

$x = \frac{7}{2} + \frac{\sqrt{33}}{2}$, $x = \frac{7}{2} - \frac{\sqrt{33}}{2}$

---

2. Compute the discriminant of the quadratic equation

$4x^2 - 20x + 25 = 0$

and discuss the nature of the roots.

discriminant is

$b^2 - 4ac$

$a = 4$, $b = -20$, $c = 25$

$b^2 - 4ac = (-20)^2 - 4(4)(25) =$

$400 - 400 = 0$.

Since the discriminant equals zero, the roots are <u>real</u> and <u>equal</u>.

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3. Solve:

$\sqrt{7 + 4x} + 9 = 0$

$\sqrt{7 + 4x} = -9$

There are <u>no solutions</u> to this equation since $\sqrt{t}$ is by definition greater than or equal to zero, for every real number t .

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4. Solve:

$x^4 - 3x^2 + 2 = 0$

Let $t = x^2$

If we let $t = x^2$, then $t^2 = x^4$

and we have:

$t^2 - 3t + 2 = 0$

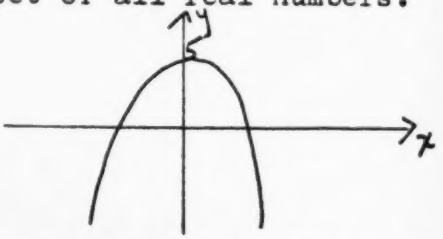
$(t - 1)(t - 2) = 0$

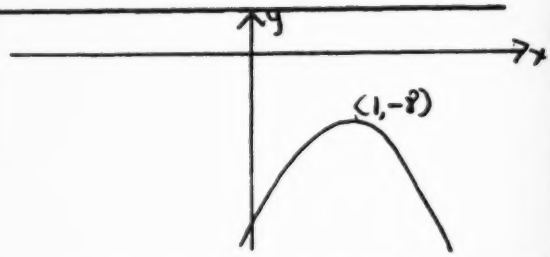
$t = 1$, $t = 2$

$x^2 = 1$, $x^2 = 2$

$x = \pm 1$, $x = \pm \sqrt{2}$

Thus $x = 1$, -1 , $\sqrt{2}$, $-\sqrt{2}$

Problem	Hint	Solution
5: Let $g(x) = 5 - 2x^2$ a) Find the domain of f . b) Find the range of f . c) Sketch the graph of f . d) Find $\frac{f(a+h) - f(a)}{h}$ for $h \neq 0$, and simplify.	The graph of $g(x)$ is a parabola opening downward (since the coefficient of x^2 is negative).	a) The domain of g is the set of all real numbers.  b) The range of g is the set of real numbers less than or equal to 5 . d) $f(a+h) = 5 - 2(a+h)^2 =$ $5 - 2(a^2 + 2ah + h^2) =$ $5 - 2a^2 - 4ah - 2h^2$ $f(a+h) - f(a) =$ $(5 - 2a^2 - 4ah - 2h^2)$ $- (5 - 2a^2) =$ $- 4ah - 2h^2 = h(-4a - 2h)$ $\frac{f(a+h) - f(a)}{h} = \frac{h(-4a - 2h)}{h}$ $= -4a - 2h \quad , h \neq 0$
6. Let $f(x) = \sqrt{2x + 3}$ and $g(x) = x^3 + 1$. Find $(f \circ g)(x)$ and $(g \circ f)(x)$.	$(f \circ g)(x) = f(g(x))$	$(f \circ g)(x) = f(g(x)) = f(x^3 + 1)$ $= \sqrt{2(x^3 + 1) + 3}$ $= \sqrt{2x^3 + 5}$ $(g \circ f)(x) = g(f(x)) = g(\sqrt{2x + 1}$ $= (\sqrt{2x + 1})^3 + 1$ $= (2x + 1)^{\frac{3}{2}} + 1$

Problem	Hint	Solution
7. Solve: $\lvert 2x - 1 \rvert < 8$	$\lvert x \rvert < a$ if and only if $-a < x < a$ where $a > 0$.	$-8 < 2x - 1 < 8$ $-8 + 1 < 2x < 8 + 1$ $-7 < 2x < 9$ $-\frac{7}{2} < x < \frac{9}{2}$
8. Solve: $-x^2 + 2x - 9 > 0$	Sketch the graph of the parabola $y = -x^2 + 2x - 9$	 The graph lies totally below the x axis, so y is always less than zero. Thus $-x^2 + 2x - 9 > 0$ has <u>no solutions</u>.
9. Solve: $\frac{x - 1}{x + 3} > 5$	$\frac{x - 1}{x + 3} - 5 > 0$	$\frac{x - 1}{x + 3} - 5 = \frac{x - 1}{x + 3} - \frac{5(x + 3)}{x + 3} =$ $\frac{x - 1 - 5(x + 3)}{x + 3} > 0$ $\frac{-4x - 16}{x + 3} > 0$ $\left(\frac{+}{-}\right)$ 0 $\left(\frac{-}{-}\right)$ U $\left(\frac{-}{+}\right)$ $-$ $+$ -4 $+$ -3 $-$ x where U means undefined. The solutions are therefore $-4 < x < -3$

Problem

10. Solve, without using Cramer's rule:

$$\begin{aligned} 2x + 5y + z &= -2 \\ -x + y + 2z &= 5 \\ 3x + y + z &= 2 \end{aligned}$$

Hint

Try to get the system into the form

$$\begin{aligned} x + by + cz &= d \\ y + ez &= f \\ z &= g \end{aligned}$$

Solution

Multiply the second equation by -1 and write it first:

$$\begin{aligned} x - y - 2z &= -5 \\ 2x + 5y + z &= -2 \\ 3x + y + z &= 2 \end{aligned}$$

Replace the second equation by itself minus twice the first.

Replace the last equation by itself minus three times the first equation:

$$\begin{aligned} x - y - 2z &= -5 \\ 7y + 5z &= 8 \\ 4y + 7z &= 17 \end{aligned}$$

Divide the second equation by 7 :

$$\begin{aligned} x - y - 2z &= -5 \\ y + \tfrac{5}{7}z &= \tfrac{8}{7} \\ 4y + 7z &= 17 \end{aligned}$$

Replace the third equation by itself minus four times the second equation:

$$\begin{aligned} x - y - 2z &= -5 \\ y + \tfrac{5}{7}z &= \tfrac{8}{7} \\ \tfrac{29}{7}z &= \tfrac{87}{7} \end{aligned}$$

Working from the bottom up:

$$z = \tfrac{7}{29} \cdot \tfrac{87}{7} = 3$$

$$y + \tfrac{5}{7}(3) = \tfrac{8}{7}$$

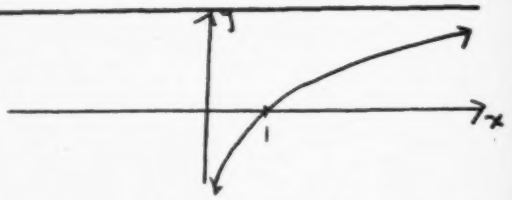
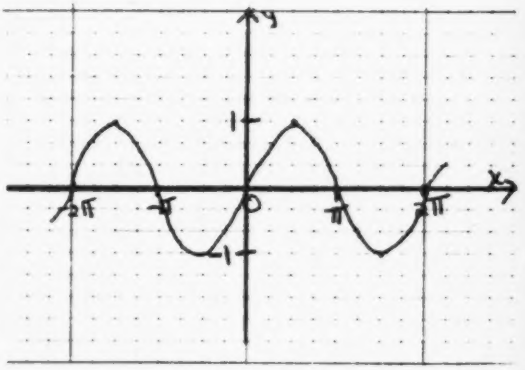
$$y = \tfrac{8}{7} - \tfrac{15}{7} = -1$$

$$x + 1 - 6 = -5$$

$$x = 0 \quad . \text{ Thus}$$

$$x = 0 \, , \; y = -1 \, , \; z = 3$$

Problem	Hint	Solution
11. Solve, if possible, using Cramer's rule: $x + 2y = 5$ $3x + 6y = 1$	Use determinants: $x = D_x/D$ $y = Dy/D$ if $D \neq 0$	$D_x = \begin{vmatrix} 5 & 2 \\ 1 & 6 \end{vmatrix} = 30 - 2 = 28$ $D = \begin{vmatrix} 1 & 2 \\ 3 & 6 \end{vmatrix} = 6-6 = 0$ Since $D_x \neq 0$ but $D = 0$, the system is <u>inconsistent</u> (no solutions). Note that if you graph the two equations, you will get two parallel lines.
12. Mixture A is 14% alchohol, and mixture B is 8% alchohol. How many liters of each should be mixed to get ten liters of a mixture that is 12% alchohol?	Let a = number of liters of mixture A b = number of liters of mixture B Find two equations in the two variables a and b.	$a + b = 10$ $.14a + .08b = (.12)(10) = 1.2$ or $a + b = 10$ $14a + 8b = 120$ From the first equation: $b = 10 - a$ $14a + 8(10 - a) = 120$ $14a + 80 - 8a = 120$ $6a = 40$ $a = \frac{40}{6} = \frac{20}{3}$ $b = 10 - \frac{20}{3} = \frac{30-20}{3} = \frac{10}{3}$
13. Write $z = r^4$ in an equivalent logarithmic expression.	The logarithim is the exponent.	$4 = \log_r z$
14. Solve a) $\log_x 16 = 2$ b) $\log_9 x = -\frac{1}{2}$	Change to exponential form.	a) $x^2 = 16$. Since the base must be positive, $x = 4$. b) $9^{-\frac{1}{2}} = x$ $\frac{1}{\sqrt{9}} = \frac{1}{3} = x$

Problem	Hint	Solution
15. Express as a single logarithm $5\log_b x + \log_b y - \log_b 6$	$\log_b(uv) = \log_b u + \log_b v$ $\log_b(u/v) = \log_b u - \log_b v$ $\log_b(u^r) = r \log_b u$	$\log_b(x^5) + \log_b(y^{\frac{1}{7}}) - \log_b 6 =$ $\log_b\left(\frac{x^5 y^{\frac{1}{7}}}{6}\right)$
16. Sketch the graph of the function $y = \log_5 x$.	This is equivalent to $5^y = x$. Fill in the table x: $\frac{1}{25}$, $\frac{1}{5}$, 1, 5, 25, 125 y: -2, -1, 0, 1, 2, 3	 x: $\frac{1}{25}$, $\frac{1}{5}$, 1, 5, 25, 125 y: -2, -1, 0, 1, 2, 3
17. Express the following angles in degrees: a) $\frac{5\pi}{3}$ b) $\frac{7\pi}{6}$ c) 3π d) $\frac{-3\pi}{4}$	Multiply the number of radians by $\frac{180^\circ}{\pi}$.	a) $\frac{5\pi}{3}\,\frac{180^\circ}{\pi} = 300^\circ$ b) $\frac{7\pi}{6}\,\frac{180^\circ}{\pi} = 210^\circ$ c) $3\pi\,\frac{180^\circ}{\pi} = 540^\circ$ d) $-\frac{3\pi}{4}\,\frac{180^\circ}{\pi} = -135^\circ$
18. Sketch the graph of the function $y = \sin x$. What is its period?		Period is 2π. 

Problem

19. Evaluate each of the following:

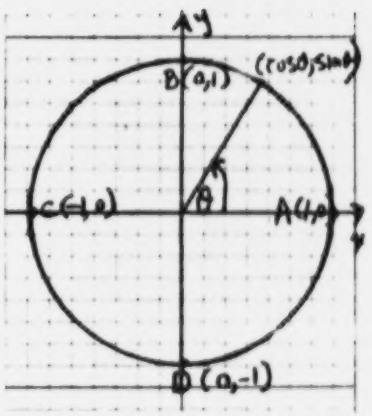
a) $\cos\left(-\frac{\pi}{2}\right)$

b) $\tan 180^\circ$

c) $\sec 2\pi$

d) $\sin 270^\circ$

Hint



Solution

a) $-\frac{\pi}{2}$ has a terminal side which intersects the unit circle at point D. Thus $\cos(-\frac{\pi}{2}) = 0$.

b) 180° intersects the unit circle at point C. Thus

$$\tan(180^\circ) = \frac{\sin(180^\circ)}{\cos(180^\circ)} = \frac{0}{-1} = 0$$

c) 2π intersects the unit circle at the point A. Thus

$$\sec 2\pi = \frac{1}{\cos 2\pi} = \frac{1}{1} = 1$$

d) 270° intersects the unit circle at point D, so $\sin 270^\circ = -1$.

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20. Evaluate each of the following:

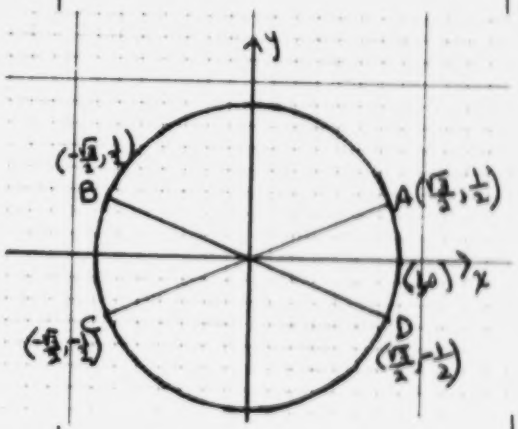
a) $\sin \frac{\pi}{6}$

b) $\cos 150^\circ$

c) $\tan \frac{7\pi}{6}$

d) $\csc 330^\circ$

e) $\cot \frac{-5\pi}{6}$



a) $\frac{\pi}{6}$ intersects the unit circle at point A, so $\sin\frac{\pi}{6} = \frac{1}{2}$.

b) 150° intersects the unit circle at point B so $\cos 150^\circ = -\frac{\sqrt{3}}{2}$.

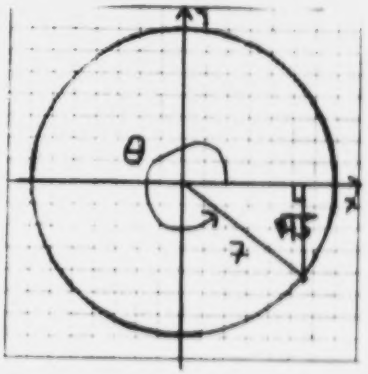
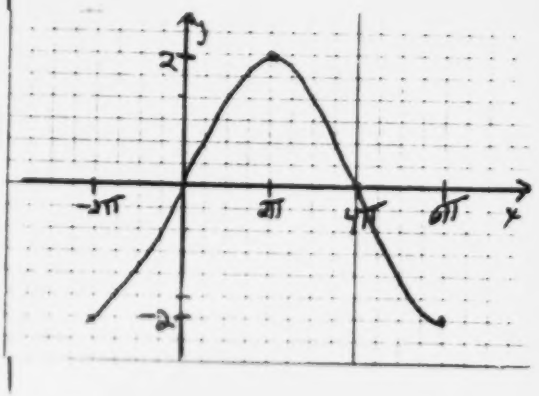
c) $\frac{7\pi}{6}$ intersects the unit circle at point C, so that

$$\tan\frac{7\pi}{6} = (\sin\tfrac{7\pi}{6})/(\cos\tfrac{7\pi}{6}) = (-1/2)/(-\sqrt{3}/2) = 1/\sqrt{3} = \sqrt{3}/3.$$

d) 330° intersects the unit circle at point D, so $\csc 330^\circ = 1/(\sin 330^\circ) = 1/-\frac{1}{2} = -2$

e) $\frac{-5\pi}{6}$ intersects the unit circle at point C, so

$$\cot(-\tfrac{5\pi}{6}) = \cos(-\tfrac{5\pi}{6})/\sin(-\tfrac{5\pi}{6}) = (-\sqrt{3}/2)/-1/2) = \sqrt{3}.$$

Problem	Hint	Solution
21. Given that $\cos\theta = \frac{2}{7}$ Find: a) $\sin\theta$ b) $\sin 2\theta$ c) $\cot\theta$ d) $\cos\frac{1}{2}\theta$ e) $\sin(\theta + \frac{\pi}{2})$ if $\frac{3\pi}{2} < \theta < 2\pi$.	$\cos\theta = \frac{2}{7} = \frac{x}{r}$ $x = 2$, $r = 7$ $x^2 + y^2 = r^2$ $4 + y^2 = 49$ $y^2 = 45$ $y = -\sqrt{45}$ (since θ is in quadrant IV). $\frac{3\pi}{4} < \frac{1}{2}\theta < \pi$ so $\frac{1}{2}\theta$ is in quadrant II , and $\cos\frac{1}{2}\theta$ is negative.  	a) $\sin\theta = \frac{y}{r} = -\frac{\sqrt{45}}{7}$ b) $\sin 2\theta = 2\sin\theta\cos\theta =$ $2\left(-\frac{\sqrt{45}}{7}\right)\frac{2}{7} = -\frac{4\sqrt{45}}{49}$ c) $\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{(2/7)}{(-\sqrt{45}/7)} =$ $= \frac{-2}{\sqrt{45}} = \frac{-2\sqrt{45}}{45}$ d) $\cos^2\frac{1}{2}\theta = \frac{1}{2}(1 + \cos\theta)$ $= \frac{1}{2}(1 + 2/7) = 9/14$. $\cos\frac{1}{2}\theta = -\sqrt{\frac{9}{14}} = \frac{-3}{\sqrt{14}} = -\frac{3\sqrt{14}}{14}$ (since $\frac{1}{2}\theta$ is in quadrant II). e) $\sin(\theta + \frac{\pi}{2}) =$ $\sin\theta\cos\frac{\pi}{2} + \cos\theta\sin\frac{\pi}{2}$ $= \sin\theta\cdot 0 + \cos\theta\cdot 1 =$ $\cos\theta = \frac{2}{7}$.
22. Let $y = -2\cos(\frac{x}{4} + \frac{\pi}{2})$ Find the amplitude, period and phase shift. Sketch one cycle of the graph.	$y = -2\cos\frac{1}{4}(x + 2\pi)$ period $= 2\pi / (1/4) =$? amplitude $= \lvert -2\rvert =$?	amplitude = 2 period = 8π phase shift = 2π (to left) 

Problem	Hint	Solution
23. Prove: a) $\frac{1}{1+\cos x} - \frac{1}{1-\cos x}$ $= -2 \cot x \csc x$ b) $\cos^4 y - \sin^4 y =$ $\cos 2y$	a) Find a common denominator b) Factor the left side	a) $\frac{1}{1+\cos x} - \frac{1}{1-\cos x} =$ $\frac{(1-\cos x)}{(1+\cos x)(1-\cos x)} - \frac{(1+\cos x)}{(1+\cos x)(1-\cos x)}$ $= \frac{1 - \cos x - 1 - \cos x}{1 - \cos^2 x}$ $= -\frac{2 \cos x}{\sin^2 x} = -2 \frac{\cos x}{\sin x} \cdot \frac{1}{\sin x}$ $= -2 \cot x \csc x$ b) $\cos^4 y - \sin^4 y =$ $(\cos^2 y - \sin^2 y)(\cos^2 Y + \sin^2 y)$ $= \cos 2y \cdot 1 = \cos 2y$
24. Express $\frac{5 + i}{6 - 2i}$ in the form a + bi.	Multiply the numerator and denominator by $6 + 2i$.	$\frac{5 + i}{6 - 2i} \cdot \frac{6 + 2i}{6 + 2i} =$ $\frac{(5 + i)(6 + 2i)}{6^2 + 2^2} =$ $\frac{30 + 10i + 6i + 2i^2}{40} =$ $\frac{30 + 16i - 2}{40} = \frac{28 + 16i}{40} =$ $\frac{28}{40} + \frac{16}{40} i = \frac{7}{10} + \frac{2}{5} i$
25. Assume x and y are real numbers. Solve for x and y $3x - y + 5yi =$ $7 + 3i$	$a + bi = c + di$ if and only if $a = c$ and $b = d$.	$3x - y = 7$ $5y = 3$ so $y = 3/5$ and $3x - (3/5) = 7$ or $3x = (38/5)$ $x = \frac{38}{15}$.

Problem	Hint	Solution
26. Let $p(x) = x^4 - x^3 + 5x^2 - 7x - 14$ a) List all possible rational roots of $p(x)$. b) Factor $p(x)$ completely c) List all the roots of $p(x)$.	If $p(x)$ is a polynomial with integer coefficients and c/d is a rational root, then c divides the constant term and d divides the leading coefficient of $p(x)$.	a) The possible rational roots are $\pm 1 , \pm 2 , \pm 7 , \pm 14.$ b) We use synthetic division to check each root: $\lfloor -1$ 1 -1 5 -7 -14 -1 2 -7 14 1 -2 7 -14 0 Thus, -1 is a root and $x+1$ is a factor of $p(x)$, with $p(x) = (x+1)(x^3-2x^2+7x-14)$. The possible rational roots of $x^3 - 2x^2 + 7x - 14$ are $\pm 1 , \pm 2 , \pm 7 , \pm 14$. Testing: $\lfloor 2$ 1 -2 7 -14 2 0 14 1 0 7 0 . Thus, 2 is a root, and $x-2$ is a factor, and $p(x) = (x+1)(x-2)(x^2+7)$ c) The roots of $p(x)$ are then $x = -1 , 2 , \sqrt{7} , -\sqrt{7}$.
27. Find the zeros with multiplicity of the polynomial $p(x) = 3x(x+2)^3(2x-5)$	Set each factor equal to zero and solve	The roots of $p(x)$ are $x = 0$ (single root) $x = -2$ (multiplicity 3) $x = \frac{5}{2}$ (single root)

Problem	Hint	Solution
28. Use the binomial theorem to expand $(x + \frac{2}{y})^5$	$(a + b)^n =$ $a^n + \binom{n}{1}a^{n-1}\,b +$ $\binom{n}{2}a^{n-2}\,b^2 + \ldots + b^n$	$(x + \frac{2}{y})^5 =$ $x^5 + \binom{5}{1}x^4\left(\frac{2}{y}\right)^1 + \binom{5}{2}x^3\left(\frac{2}{y}\right)^2 +$ $\binom{5}{3}x^2\left(\frac{2}{y}\right)^3 + \binom{5}{4}x^1\left(\frac{2}{y}\right)^4 + \left(\frac{2}{y}\right)^5 =$ $x^5 + 5x^4 \cdot \frac{2}{y} + 10x^3 \cdot \frac{4}{y^2} +$ $10x^2 \cdot \frac{8}{y^3} + 5x\,\frac{16}{y^4} + y^5$ $= x^5 + 10\,\frac{x^4}{y} + 40\,\frac{x^3}{y^2} +$ $80\,\frac{x^2}{y^3} + 80\,\frac{x}{y^4} + \frac{32}{y^5}$
29. An arc 6 ft long on a circle of radius 24 ft subtends an angle of how many radians? How many degrees are in this angle to the nearest degree?	$s = \theta\, r$ when θ is measured in radians. $1 \text{ rdn} = \frac{180^\circ}{\pi} = 57.3^\circ$	Since $s = \theta\, r$ and $s = 6$ ft, $r = 24$ ft, $\theta = \frac{s}{r} = \frac{6}{24} = \frac{1}{4}$ radian $\theta = \frac{1}{4}(57.3^\circ) = 14.325^\circ \approx 14^\circ$

REVIEW SHEET # 3

1. Solve by completing the square: $2x^2 - 12x + 1 = 0$.

2. Find a quadratic equation whose roots are -2 and $\frac{3}{5}$.

3. Solve: $\sqrt{1 - 3x} = 2x$.

4. Solve: $x - 8\sqrt{x} - 7 = 0$.

5. Let $h(x) = \dfrac{4x}{5 - |x + 2|}$

   a) Find the domain of h .

   b) Find $h(0)$, $h(4)$, and $h(-4)$.

6. Graph the function $f(x) = |x|$.
   Is it an odd function, an even function, or neither?

7. Solve: $|4 - 3x| \geq 2$.

8. Solve: $x^2 - 5x + 4 \leq 0$.

9. Solve: $(x + 1)(x + 4)(x - 7) \leq 0$.

10. Evaluate the determinant $\begin{vmatrix} -2 & 1 & 3 \\ 1 & 0 & 4 \\ -1 & -3 & 5 \end{vmatrix}$.

11. Use Cramer's Rule to express y as the quotient of two determinants.

$$\begin{aligned} x + 2y + 3z &= 7 \\ -x + y + 5z &= 0 \\ 11x - 4y - z &= -1 \end{aligned}$$

12. Solve, if possible

$$\begin{aligned} x + y + z &= 4 \\ -2x - y - 7z &= 0 \\ 3x - y + 23z &= 6 \end{aligned}$$

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13. A man rows 18 miles downstream in 2 hours. He finds that it takes him 6 hours to row back. Find his rate in still water and the rate of the stream.

14. Write an exponential expression equivalent to $\log_2 y = b$.

15. Solve for x : a) $\log_5 x = 0$

b) $\log_x 2 = \frac{1}{3}$.

16. Express as a single logarithm and simplify, if possible:

$$\log_a(x^2 + 4x + 3) - \log_a(x + 3) \quad .$$

17. Sketch the graph of the function $y = \left(\frac{1}{3}\right)^x$.

18. Express the following angles in radians:

a) 180° b) 270° c) -30° d) 120° e) 25°

19. Sketch the graph of the function $y = \tan x$. What is its period?

20. Evaluate each of the following:

a) $\tan \frac{\pi}{2}$ b) $\sec(-180^\circ)$ c) $\sin 2\pi$ d) $\cos(-270^\circ)$

21. Evaluate each of the following:

a) $\tan 60^\circ$ b) $\sin \frac{2\pi}{3}$ c) $\cos 240^\circ$ d) $\cot \frac{5\pi}{3}$ e) $\sec\left(-\frac{5\pi}{3}\right)$

22. Given that $\tan \theta = \frac{3}{4}$, where $\pi < \theta < \frac{3\pi}{2}$, find

a) $\sin \theta$ b) $\cos \theta$ c) $\sin 2\theta$ d) $\cos \frac{1}{2}\theta$ e) $\tan(\theta + \pi)$.

23. Consider $y = 3\sin(2x - \frac{\pi}{4})$.

Find the amplitude, period, and phase shift. Then sketch one cycle of the graph.

24. Prove:

a) $\tan y (\tan y + \cot y) = \sec^2 y$

b) $\frac{1 + \cos x}{\sin x} + \frac{\sin x}{\cos x} = \frac{4}{\sin 2x}$

25. Express each of the following in the form $a + bi$:

a) $(6 + 2i)^2$ b) $i^{11} - i^{12}$.

26. Solve: $x^2 - 2x + 10 = 0$.

27. In the expression for $(3 - \sqrt{x})^7$

a) Find the fourth term.

b) Find the coefficient of x^2 .

28. In a circle, an arc 50 cm long subtends a central angle of 75° . How long is the radius?

29. Let $p(x) = 2x^4 - 9x^3 + 6x^2 + 11x - 6$.

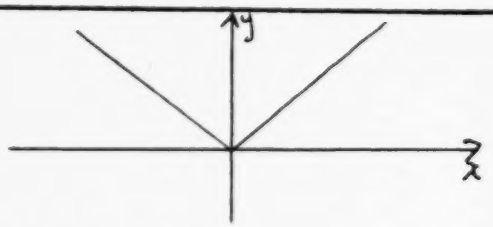
a) Factor $p(x)$ completely.

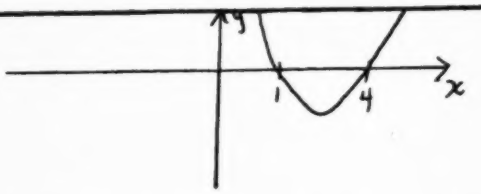
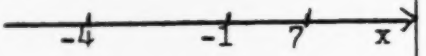
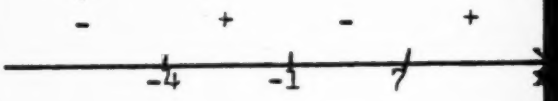
b) Find all of the zeros of $p(x)$.

30. Find a polynomial of lowest degree with rational coefficients that has $1 - \sqrt{2}$ and $3 + i$ as two of its roots.

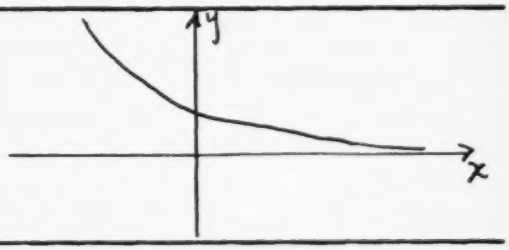
REVIEW SHEET #3 - SOLUTIONS

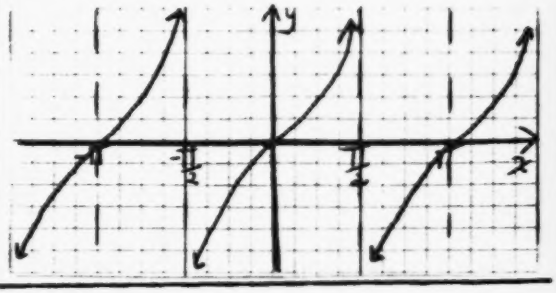
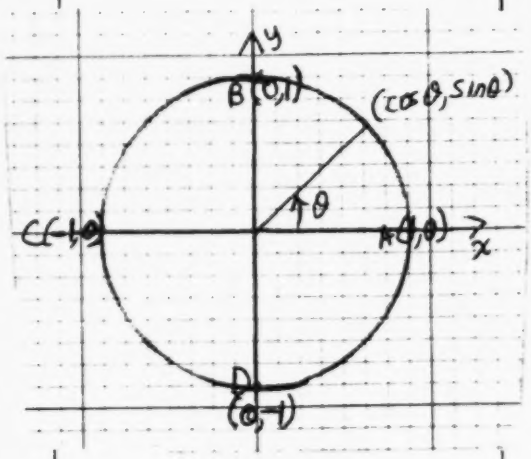
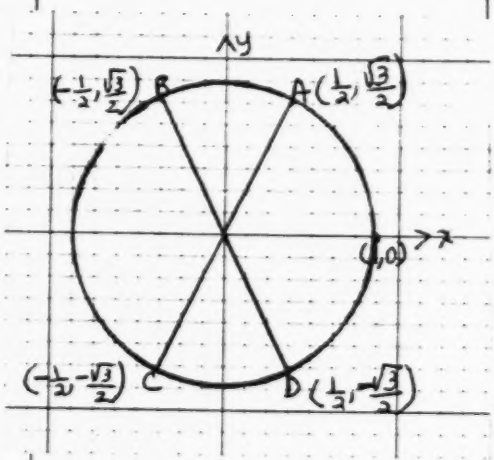
Problem	Hint	Solution
1. Solve by completing the square: $2x^2 - 12x + 1 = 0$	Divide both sides of the equation by 2 to make the coefficient of x^2 equal to 1.	$2x^2 - 12x + 1 = 0$ $x^2 - 6x + \frac{1}{2} = 0$ $x^2 - 6x + __ = -\frac{1}{2} + __$ $x^2 - 6x + 9 = -\frac{1}{2} + 9$ $(x - 3)^2 = \frac{17}{2}$ $x - 3 = \pm\sqrt{\frac{17}{2}} = \pm\frac{\sqrt{17}}{\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}} = \pm\frac{\sqrt{34}}{2}$ $x = 3 \pm \frac{\sqrt{34}}{2}$, $3 - \frac{\sqrt{34}}{2}$
2. Find a quadratic equation whose roots are -2 and $\frac{3}{5}$.	$(x - ?)(x - ?) = 0$	Since -2 is a root, $x - (-2) = x + 2$ is a factor of the quadratic. Since $\frac{3}{5}$ is a root, $x - \frac{3}{5}$ is a factor of the quadratic. Thus, $(x + 2)(x - \frac{3}{5}) = 0$ or $x^2 + \frac{7}{5}x - \frac{6}{5} = 0$ or $5x^2 + 7x - 6 = 0$ is the equation.
3. Solve: $\sqrt{1 - 3x} = 2x$	Square both sides. Be sure to check your answers.	$(1 - 3x)^2 = (2x)^2$ $1 - 3x = 4x^2$ $4x^2 + 3x - 1 = 0$ $(4x - 1)(x + 1) = 0$ $x = \frac{1}{4}$, $x = -1$ check $x = \frac{1}{4}$: $\sqrt{1 - \frac{3}{4}} = \sqrt{\frac{1}{4}} = \frac{1}{2} = 2(\frac{1}{4})$ ✓ check $x = -1$: $\sqrt{1 - 3(-1)} = \sqrt{4} = 2 \neq 2(-1)$ so -1 is extraneous. The answer is $x = \frac{1}{4}$

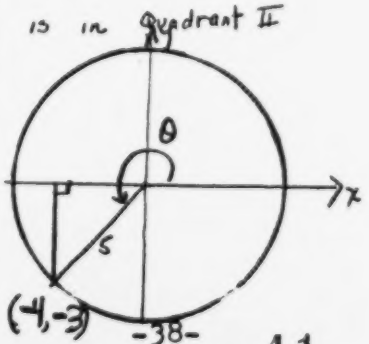
Problem	Hint	Solution
4. Solve $x - 8\sqrt{x} + 7 = 0$	Let $u = \sqrt{x}$	If $u = \sqrt{x}$, then $u^2 = x$, and we have $u^2 - 8u + 7 = 0$ $(u - 7)(u - 1) = 0$ $u = 7$, $u = 1$, so $\sqrt{x} = 7$, $\sqrt{x} = 1$ $x = 49$, $x = 1$. Checking $x = 49$: $49 - 8\sqrt{49} + 7 = 0$ ✓ Checking $x = 1$: $1 - 8\sqrt{1} + 7 = 0$ ✓
5. Let $h(x) = \frac{4x}{5 - \lvert x+2 \rvert}$ a) Find the domain of h . b) Find h(0) , h(4), h(-4).	The denominator of h(x) cannot be zero.	a) If $5 - \lvert x + 2 \rvert = 0$, then $5 = \lvert x + 2 \rvert$ $x + 2 = 5$ or $x + 2 = -5$ $x = 3$ or $x = -7$. Thus the domain of h is all real numbers except 3 and -7 . b) $h(0) = \frac{4(0)}{5 - \lvert 0+2 \rvert} = \frac{0}{3} = 0$ $h(4) = \frac{4(4)}{5 - \lvert 4+2 \rvert} = \frac{16}{5-6} = -16$ $h(-4) = \frac{4(-4)}{5 - \lvert -4+2 \rvert} = \frac{-16}{5-2} = -\frac{16}{3}$
6. Graph the function $f(x) = \lvert x \rvert$. Is it odd, even or neither?	plot points: x | 0 | 1 | -1 | 2 | -2 y | | | | |	 f is even, because $f(-x) = \lvert -x \rvert = \lvert x \rvert = f(x)$

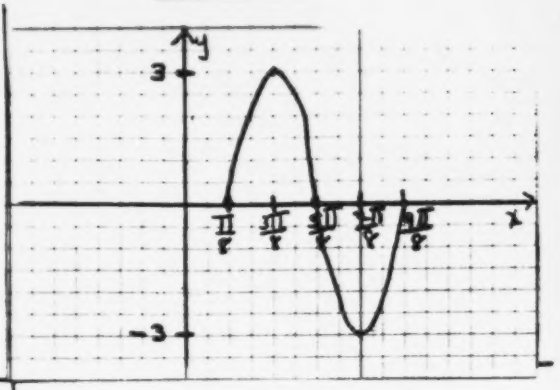
Problem	Hint	Solution
7. Solve: $\lvert 4 - 3x \rvert \geq 2$	For $a > 0$, $\lvert x \rvert \geq a$ is equivalent to $x \leq -a$ or $x \geq a$	$4 - 3x \geq 2$ or $4 - 3x \leq -2$ $-3x \geq -2$ or $-3x \leq -6$ $x \leq \frac{2}{3}$ or $x \geq 2$
8. Solve: $x^2 - 5x + 4 \leq 0$	Sketch the graph of $y = x^2 - 5x + 4 = (x - 1)(x - 4)$ and see when $y \leq 0$, i. e. when the graph lies below the x axis.	 We see from the graph that $y \leq 0$ when $1 \leq x \leq 4$.
9. Solve: $(x+1)(x+4)(x-7) \leq 0$	Test a point in each interval on this number line: 	 Thus the solution is $x \leq -4$ or $-1 \leq x \leq 7$.
10. Evaluate the determinant $\begin{vmatrix} -2 & 1 & 3 \\ 1 & 0 & 4 \\ -1 & -3 & 5 \end{vmatrix}$	Expand using the second row.	$-1\begin{vmatrix} 1 & 3 \\ -3 & 5 \end{vmatrix} + 0 + -4\begin{vmatrix} -2 & 1 \\ -1 & -3 \end{vmatrix}$ $= -1(5 + 9) - 4(6 + 1) =$ $-14 - 28 = -42$
11. Use Cramer's Rule to express y as the quotient of determinants: $x + 2y + 3z = 7$ $-x + y + 5z = 0$ $11x - 4y - z = -1$	$y = \frac{D_y}{D}$	$y = \dfrac{\begin{vmatrix} 7 & 2 & 3 \\ 0 & 1 & 5 \\ -1 & -4 & -1 \end{vmatrix}}{\begin{vmatrix} 1 & 2 & 3 \\ -1 & 1 & 5 \\ 11 & -4 & -1 \end{vmatrix}}$

Problem	Hint	Solution
12. Solve, if possible $x + y + z = 4$ $-2x - y - 7z = 0$ $3x - y + 23z = 6$	Try to get the system into the form $x + ay + bz = c$ $dy + ez = f$ $z = g$	Replace the second equation by itself plus twice the first. Replace the last equation by itself minus three times the first. $x + y + z = 4$ $y - 5z = 8$ $-4y + 20z = -6$ Replace the last equation by itself plus four times the second. $x + y + z = 4$ $y - 5z = 8$ $0 = 26$ Since this last statement is clearly false, the equations are <u>inconsistent</u>.
13. A man rows 18 miles downstream in 2 hours. He finds that it takes him 6 hours to row back. Find his rate in still water and the rate of the spring.	Let x = rate of man in still water y = rate of man in spring note: rate time = distance so $\text{rate} = \frac{\text{distance}}{\text{time}}$	rate of travel downstream: $\frac{18}{2}$ m.p.h. = 9 m.p.h. rate of travel upstream: $\frac{18}{6}$ m.p.h. = 3 m.p.h. so $x + y = 9$ $x - y = 3$ $2x = 12$ $x = 6$ m.p.h. $y = 3$ m.p.h.
14. Write an exponential expression equivalent to $\log_2 y = b$	The logarithm is the exponent.	$2^b = y$

Problem	Hint	Solution
15. Solve for x: a) $\log_5 x = 0$ b) $\log_x 2 = \frac{1}{3}$	Write it in exponential form.	a) $5^0 = x$ $1 = x$ b) $x^{\frac{1}{3}} = 2$ $x = 2$ $x = 2^3 = 8$
16. Express as a single logarithm and simplify, if possible: $\log_a(x^2 + 4x + 3) - \log_a(x + 3)$	$\log_a(u) - \log_a(v) = \log_a(u/v)$	$\log_a(x^2 + 4x + 3) - \log_a(x + 3) =$ $\log_a \frac{x^2 + 4x + 3}{x + 3} =$ $\log_a \frac{(x + 1)(x + 3)}{x + 3} =$ $\log_a (x + 1)$
17. Sketch the graph of the function $y = \left(\frac{1}{3}\right)^x$	Fill in the table x: -3 -2 -1 0 1 2 y:	
18. Express the following angles in radians: a) 180° b) 270° c) -30° d) 120° e) 25°	Multiply the number of degrees by $\frac{\pi}{180^\circ}$	a) $180^\circ \times \frac{\pi}{180^\circ} = \pi$ b) $270^\circ \times \frac{\pi}{180^\circ} = \frac{3\pi}{2}$ c) $-30^\circ \times \frac{\pi}{180^\circ} = -\frac{\pi}{6}$ d) $120^\circ \times \frac{\pi}{180^\circ} = \frac{2\pi}{3}$ e) $25^\circ \times \frac{\pi}{180^\circ} = \frac{5\pi}{36}$

Problem	Hint	Solution
19. Sketch the graph of the function $y = \tan x$. What is its period?	$\tan(x + ?) = \tan x$	Period : π 
20. Evaluate each of the following: a) $\tan \frac{\pi}{2}$ b) $\sec(-180^\circ)$ c) $\sin 2\pi$ d) $\cos(-270^\circ)$		a) $\frac{\pi}{2}$ intersects the unit circle at point B. Thus $\tan \frac{\pi}{2}$ is undefined because $\cos \frac{\pi}{2} = 0$. b) -180° intersects the unit circle at point C. Thus $\sec 180^\circ = \frac{1}{\cos 180^\circ} = \frac{1}{-1} = -1$ c) 2π intersects the unit circle at the point A. Thus, $\sin 2\pi = 0$. d) -270° intersects the unit circle at point B. Thus, $\cos(-270^\circ) = 0$.
21. Evaluate each of the following: a) $\tan 60^\circ$ b) $\sin \frac{2\pi}{3}$ c) $\cos 240^\circ$ d) $\cot \frac{5\pi}{3}$ e) $\sec \frac{-5\pi}{3}$		a) 60° intersects the unit circle at point A. Thus, $\tan 60^\circ = \frac{\sin 60^\circ}{\cos 60^\circ} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$ b) $\frac{2\pi}{3}$ intersects the unit circle at point B. Thus, $\sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$.

Problem	Hint	Solution
21. continued		c) 240° intersects the unit circle at point C. Thus, $\cos 240^\circ = -\frac{1}{2}$. d) $\frac{5\pi}{3}$ intersects the unit circle at point D. Thus, $\cot \frac{5\pi}{3} = (\cos \frac{5\pi}{3})/(\sin \frac{5\pi}{3})$ $= \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{-1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$ e) $-\frac{5\pi}{3}$ intersects the unit circle at point A . Thus, $\sec -\frac{5\pi}{3} = \frac{1}{\cos -\frac{5\pi}{3}} = \frac{1}{\frac{1}{2}} = 2$
22. Given than $\tan\theta = \frac{3}{4}$, where $\pi < \theta < \frac{3\pi}{2}$ Find: a) sin θ b) cos θ c) sin 2θ d) cos $\frac{1}{2}\theta$ e) tan (θ + π)	$\tan\theta = \frac{x}{y} = \frac{3}{4} = \frac{-3}{-4}$ Since θ is in quadrant III , x = -3 , y = - 4 $x^2 + y^2 = r^2$ $r = \sqrt{9 + 16} = 5$ Since $\pi < \theta < \frac{3\pi}{2}$ $\frac{\pi}{2} < \frac{1}{2}\theta < \frac{3\pi}{4}$ is in Quadrant II  	a) $\sin\theta = \frac{y}{r} = -\frac{3}{5}$ b) $\cos\theta = \frac{x}{r} = -\frac{4}{5}$ c) $\sin 2\theta = 2 \sin\theta \cos\theta = 2 \frac{-3}{5} \frac{-4}{5} = \frac{24}{25}$ d) $\cos^2 \frac{1}{2}\theta = \frac{1}{2}(1 + \cos\theta) = \frac{1}{2}(1 + \frac{4}{5}) = \frac{1}{2}.\frac{9}{5} = \frac{9}{10}$ $\cos \frac{1}{2}\theta = -\sqrt{\frac{9}{10}} = \frac{-3}{\sqrt{10}} = \frac{-3\sqrt{10}}{10}$ e) $\tan(\theta + \pi) = \tan\theta = \frac{3}{4}$

Problem	Hint	Solution
23. Let $y = 3\sin(2x - \frac{\pi}{4})$ Find the amplitude, period and phase shift. Then sketch one cycle of the graph.	$y = 3\sin 2(x - \frac{\pi}{8})$ amplitude: 3 period: $\frac{2\pi}{2} = \pi$ phase shift: $\frac{\pi}{8}$ (to right)	
24. Prove a) $\tan y(\tan y + \cot y) = \sec^2 y$ b) $\frac{1+\cos x}{\sin x} + \frac{\sin x}{\cos x} = \csc x + \csc x \sec x$	a) multiply out on the left side b) Find a common denominator on the left side.	a) $\tan y(\tan y + \cot y) =$ $\tan^2 y + \tan y \cot y =$ $\tan^2 y + 1 = \sec^2 y$ b) $\frac{1 + \cos x}{\sin x} + \frac{\sin x}{\cos x} =$ $\frac{(1 + \cos x)\cos x + \sin x \sin x}{\sin x \cos x}$ $= \frac{\cos x + (\cos^2 x + \sin^2 x)}{\sin x \cos x}$ $= \frac{\cos x + 1}{\sin x \cos x} =$ $= \frac{\cos x}{\sin x \cos x} + \frac{1}{\sin x \cos x}$ $= \frac{1}{\sin x} + \frac{1}{\sin x} \frac{1}{\cos x}$ $= \csc x + \csc x \sec x$

Problem	Hint	Solution
25. Express in the form a + bi a) $(6 + 2i)^2$ b) $i^{11} - i^{12}$	To find i^k, divide k by 4. Let r be the remainder. If $\frac{r}{0}$ then $\frac{i^k}{1}$ 1 i 2 -1 3 $-i$	a) $(6 + 2i)(6 + 2i) =$ $36 + 24i + 4i^2 =$ $36 + 24i + 4(-1) =$ $32 + 24i$ b) $i^{11} = -i$ $i^{12} = 1$ $i^{11} - i^{12} = -i - 1 = -1 - i$
26. Solve: $x^2 - 2x + 10 = 0$	Use the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	$a = 1$, $b = -2$, $c = 10$ $x = \frac{2 \pm \sqrt{4 - 4(10)}}{2}$ $= \frac{2 \pm \sqrt{-36}}{2} = \frac{2 \pm \sqrt{36}\sqrt{-1}}{2} =$ $= \frac{2 \pm 6i}{2} = 1 \pm 3i$
27. In the expression for $(3 - \sqrt{x})^7$ a) Find the fourth term b) Find the coefficient of x^2.	The k+1 term is $\binom{7}{k} 3^{7-k} (-\sqrt{x})^k$	a) The fourth term is $\binom{7}{3} 3^4 (-\sqrt{x})^3 = \frac{7\ 6\ 5}{4\ 3\ 2\ 1} 81 \cdot -x^{\frac{3}{2}}$ $= -2835\, x^{\frac{3}{2}}$ b) The term containing x^2 is $\binom{7}{4} 3^3 (-\sqrt{x})^4 =$ $\frac{7\ 6\ 5}{3\ 2\ 1} 27\, x^2 = 945\, x^2$. The coefficient of x^2 is 945.
28. In a circle, an arc 50 cm long subtends a central angle of 75°. How long is the radius?	$s = \theta r$ when θ is measured in radians	$s = \theta r$ $\theta = 75^\circ = 75^\circ \times \frac{\pi}{180^\circ} = \frac{5\pi}{12}$ rdn $r = \frac{s}{\theta} = \frac{50}{\frac{5\pi}{12}} = 50 \times \frac{12}{5\pi} = \frac{120}{\pi}$ $= 38$ cm

Problem	Hint	Solution
29. Let $p(x)=2x^4-9x^3+6x^2+11x-6$ a) Factor p(x) completely. b) Find all the zeros of p(x).	The possible rational roots of p(x) are ± 1 , ± 2 , ± 3, ± 6 , $\pm \frac{1}{2}$, $\pm \frac{3}{2}$ We use synthetic division to test each one.	$\lfloor 2$ 2 -9 6 11 -6 4 -10 -8 6 2 -5 -4 3 0 2 is a root and x-2 is a factor $p(x) = (x-2)(2x^3-5x^2-4x+3)$ The possible rational roots of $2x^3-5x^2-4x+3$ are ± 1 , ± 3 , $\pm \frac{3}{2}$. $\lfloor 3$ 2 -5 -4 3 6 3 -3 2 1 -1 0 3 is a root, and x-3 a factor. $p(x) = (x-2)(x-3)(2x^2+x-1) =$ $(x-2)(x-3)(2x-1)(x+1)$ The zeros of p(x) are $x = 2 , 3 , \frac{1}{2} , -1$.
30. Find a polynomial of lowest degree with rational coefficients, that has $1-\sqrt{2}$ and $3+i$ as two of its roots.	In order for a polynomial with rational coefficients to have irrational and/or complex roots, these roots must occur in conjugate pairs.	The roots are $1-\sqrt{2}$, $1+\sqrt{2}$, $3+1$, $3-1$. The polynomial is $(x-(1-\sqrt{2}))(x-(1+\sqrt{2}))(x-(3+1)$ $(x-(3-1))$.

REVIEW SHEET # 4

1. Write an exponential expression equivalent to $\log_5 y = c$.

2. Sketch the graph of the function $y = \cot x$. What is its period?

3. Solve by completing the square: $5x^2 + x - 3 = 0$.

4. Let $f(x) = \begin{cases} \frac{2}{x^2+16} & , \quad x \leq -1 \\ 6 & , \quad -1 < x < 4 \\ 9 - 3x & , \quad x \geq 4 \end{cases}$

   a) Find the domain of f .

   b) Fine $f(-4)$, $f(0)$, $f(5)$.

5. Solve: $x^{-2} + 9x^{-1} = 0$.

6. Solve: $\sqrt{2x - 1} - \sqrt{x - 1} = 1$.

7. Evaluate the determinant $\begin{vmatrix} 1 & 5 & -3 \\ 0 & -2 & 1 \\ -2 & -1 & 1 \end{vmatrix}$

8. Express the following angles in degrees:

   a) $\frac{\pi}{5}$ b) 2π c) $-\frac{\pi}{3}$ d) $\frac{9\pi}{4}$

9. Prove

   a) $\cos^2 x \sin^2 x + \cos^4 x = \cos^2 x$

   b) $\frac{\cos(x+y)}{\cos x \cos y} = 1 - \tan x \tan y$

10. Find the middle term of $(a + \frac{1}{3} b)^8$.

11. Solve without using Cramer's Rule:

$$x + 2y + 3z = 0$$

$$x + y + z = 0$$

$$5x + 7y + 9z = 0$$

12. Let $p(x) = x^4 + x^3 + 10x^2 + 9x + 9$. Given that $3i$ is a root of this polynomial, find the other roots and factor $p(x)$.

13. Compute the discriminant of the quadratic equation $x^2 + 4x - 2 = 0$ and discuss the nature of the roots.

14. Given that $\sin\theta = -\frac{4}{5}$ and $0 < \theta < \frac{\pi}{2}$, find

a) $\sin(\theta + 45^\circ)$ b) $\cos(\theta - \frac{\pi}{3})$ c) $\tan(\theta + \frac{\pi}{6})$

15. Graph the function $f(x) = 2^x$. Is it even, odd, or neither?

16. Solve: $\frac{(4 - 2x)}{(x + 6)(x - 3)} \geq 0$

17. One day a store sold 60 blouses. Blue ones sold for \$10.50 each and white ones sold for \$9.75 each. In all \$615 worth of blouses were sold. How many of each color were sold?

18. Solve: $\log_a x^3 = 3$.

19. Assume that x and y are real numbers and solve for x and y:

$$x + 3y + 2i = 6 + (2x + y)i \quad .$$

20. Evaluate each of the following:

a) $\csc\left(-\frac{\pi}{2}\right)$ b) $\sin(-\pi)$ c) $\cos 360^\circ$ d) $\csc\frac{3\pi}{2}$

21. Solve $|1 - 9x| < -6$

22. Solve: $x^2 - 3x - 10 \geq 0$.

23. Express the following in terms of logarithms of x , y , and z:

$$\log_a \sqrt[9]{\frac{xy}{z^3}}$$

24. Evaluate each of the following:

a) $\sin 405^\circ$ b) $\tan \frac{17\pi}{6}$ c) $\csc(-\frac{8\pi}{3})$ d) $\cot 45^\circ$

25. Let $y = -\cos(\frac{x}{3} + \pi)$. Find the amplitude, period and phase shift. Sketch one cycle of the graph.

26. Express each of the following in the form a + bi:

a) $\frac{\sqrt{5} + 3i}{\sqrt{5} - 3i}$ b) $i^{40} - i^{51} - i^{122} + i^{53}$.

27. Solve: $\log_2 x + \log_2 (x + 2) = 3$.

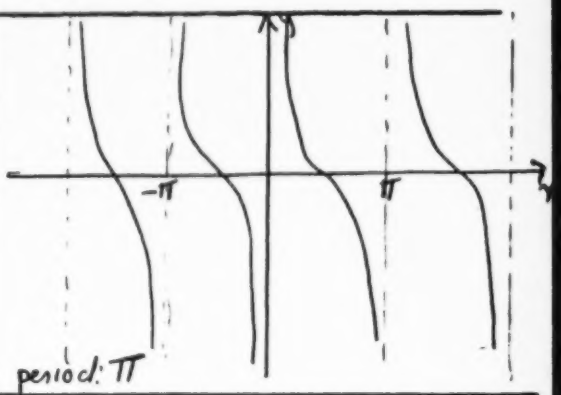
28. A subcompact car uses a tire whose radius is 14 in.

a) How far has the car moved when the tire has completed one rotation?
b) How many rotations are completed when the tire has traveled one mile?

29. Find all the roots, with multiplicities of

$$p(x) = (x^2 - 6x + 9)(x^2 - 25) .$$

REVIEW SHEET #4 - SOLUTIONS

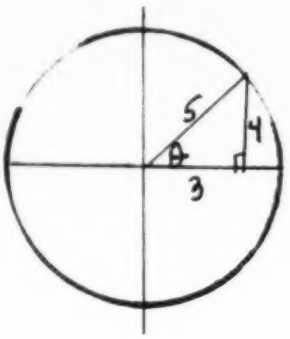
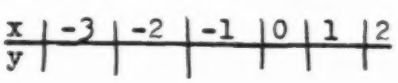
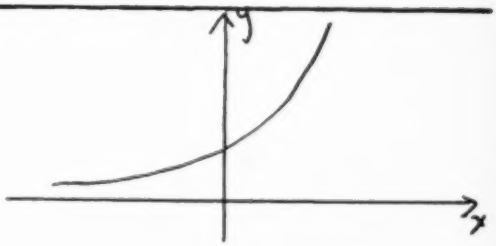
Problem	Hint	Solution
1. Write an exponential expression equivalent to $\log_5 y = c$.	The logarithm is the exponent.	$5^c = y$
2. Sketch the graph of the function $y = \cot x$. What is its period?	$\cot(x + ?) = \cot x$	
3. Solve by completing the square: $5x^2 + x - 3 = 0$	Divide both sides of the equation by 5 to make the coefficient of x^2 equal to 1 .	$5x^2 + x - 3 = 0$ $x^2 + \frac{1}{5}x - \frac{3}{5} = 0$ $x^2 + \frac{1}{5}x + __ = \frac{3}{5} + __$ $x^2 + \frac{1}{5}x + \frac{1}{100} = \frac{3}{5} + \frac{1}{100}$ $(x + \frac{1}{10})^2 = \frac{61}{100}$ $x + \frac{1}{10} = \pm\sqrt{\frac{61}{100}} = \pm\frac{\sqrt{61}}{10}$ $x = -\frac{1}{10} \pm \frac{\sqrt{61}}{10}$
4. Let $f(x) = \begin{cases} \frac{2}{x^2+16}, & x \le -1 \\ 6, & -1 < x < 4 \\ 9-3x, & x \ge 4 \end{cases}$ a) Find the domain of f. b) Find f(-4), f(0), f(5)	To find the domain, put together the domain in each piece.	a) Domain: all real numbers b) $f(0) = 6$ since 0 is between -1 and 4 $f(-4) = \frac{2}{(-4)^2+16} = \frac{2}{32} = \frac{1}{16}$ since -4 is less than -1. $f(5) = 9-3(5)$ since $5 \ge 4$. $= -6$

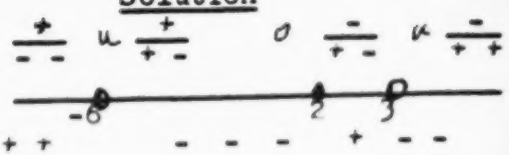
Problem	Hint	Solution
5. Solve: $x^{-2} + 9x^{-1} = 0$	Let $t = x^{-1}$	If $t = x^{-1}$, then $t^2 = (x^{-1})^2 = x^{-2}$ so our equation becomes $t^2 + 9t = 0$, or $t(t + 9) = 0$, so $t = 0$ or $t = -9$ $x^{-1} = \frac{1}{x} = 0$, which is impossible , or $x^{-1} = \frac{1}{x} = -9$, so $x = -\frac{1}{9}$
6. Solve: $\sqrt{2x - 1} - \sqrt{x-1} = 1$	Isolate one radical: $\sqrt{2x-1} = 1 + \sqrt{x - 1}$ then square both sides.	$\sqrt{2x - 1} = 1 + \sqrt{x - 1}$ $(\sqrt{2x - 1})^2 = (1 + \sqrt{x - 1})^2$ $2x - 1 = 1 + 2\sqrt{x - 1} + x - 1$ $x - 1 = 2\sqrt{x - 1}$ squaring again: $(x - 1)^2 = (2\sqrt{x - 1})^2$ $x^2 - 2x + 1 = 4(x-1) = 4x-4$ $x^2 - 6x + 5 = 0$ $(x - 1)(x - 5) = 0$ $x = 1$, $x = 5$ checking our answers: $\sqrt{2(1)-1} - \sqrt{1-1} = 1 - 0 = 1$ so $x = 1$ is a solution. $\sqrt{2(5)-1} - \sqrt{5-1} = \sqrt{9} - \sqrt{4} = 3-2 = 1$ so 5 is a solution. Answers: $x = 1$, 5 .

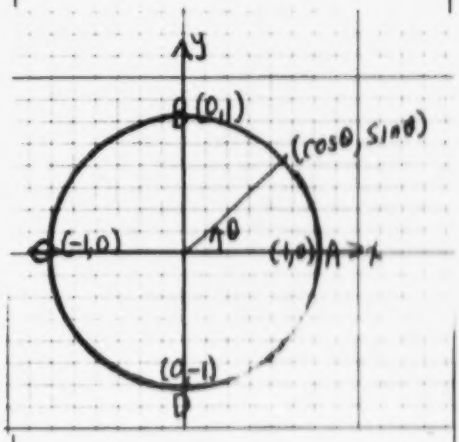
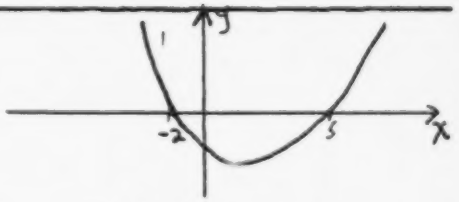
Problem	Hint	Solution
7. Evaluate the determinant $\begin{vmatrix} 1 & 5 & -3 \\ 0 & -2 & 1 \\ -2 & -1 & 1 \end{vmatrix}$	Expand about the first column.	$1\begin{vmatrix} -2 & 1 \\ -1 & 1 \end{vmatrix} - 0 - ?\begin{vmatrix} 5 & -3 \\ -2 & 1 \end{vmatrix}$ $= (-2 + 1) + 0 \quad -2(5 - 6) =$ $-1 \quad -2(-1) = -1 + 2 = 1$
8. Express the following angles in degrees: a) $\frac{\pi}{3}$ b) 2π c) $-\frac{\pi}{5}$ d) $\frac{9\pi}{4}$	Multiply the number of radians by $\frac{180^\circ}{\pi}$.	a) $\frac{\pi}{3} \cdot \frac{180^\circ}{\pi} = 60^\circ$ b) $2\pi \cdot \frac{180^\circ}{\pi} = 360^\circ$ c) $-\frac{\pi}{5} \cdot \frac{180^\circ}{\pi} = -36^\circ$ d) $\frac{9\pi}{4} \cdot \frac{180^\circ}{\pi} = 405^\circ$
9. Prove the identity a) $\cos^2 x \sin^2 x + \cos^4 x = \cos^2 x$ b) $\frac{\cos(x+y)}{\cos x \cos y} = 1 - \tan x \tan y$	Factor on the left side $\cos(x + y) = ?$	a) $\cos^2 x \sin^2 x + \cos^4 x =$ $\cos^2 x(\sin^2 x + \cos^2 x) =$ $\cos^2 x\,(1) = \cos^2 x$ b) $\frac{\cos(x + y)}{\cos x \cos y} =$ $\frac{\cos x \cos y - \sin x \sin y}{\cos x \cos y}$ $= \frac{\cos x \cos y}{\cos x \cos y} - \frac{\sin x \sin y}{\cos x \cos y}$ $= 1 - \tan x \tan y$
10. Find the middle term of $(a + \frac{1}{3} b)^8$	The k+1 term is $\binom{8}{8-k} a^{8-k} (\frac{1}{3} b)^k$	There are 9 terms in the expansion. The middle term is thus the fifth: $\binom{8}{4} a^4 \times (\frac{1}{3} b)^4 =$ $\frac{8\,7\,6\,5\,4\,3\,2\,1}{4\,3\,2\,1\,4\,3\,2\,1} a^4 \frac{1}{81} b^4$ $= 70\, a^4 \frac{1}{81} b^4 = \frac{70}{81} a^4 b^4$

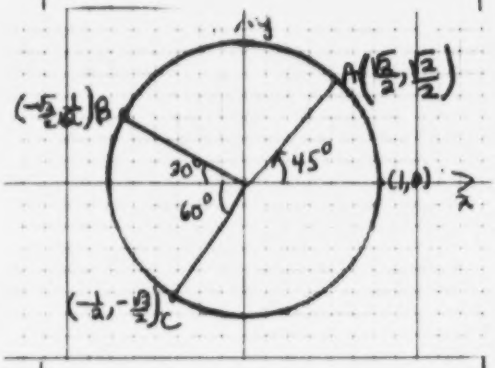
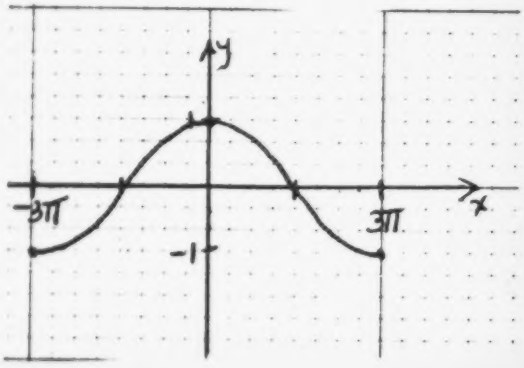
Problem	Hint	Solution
11. Solve without using Cramer's Rule: $x + 2y + 3z = 0$ $x + y + z = 0$ $5x + 7y + 9z = 0$	Try to get the system into the form $x + ay + bz = c$ $y + dz = e$ $z = f$	Replace the second equation by itself minus the first: Replace the third equation by itself minus 5 times first. $x + 2y + 3z = 0$ $-y - 2z = 0$ $-3y - 6z = 0$ Multiply the second equation by -1: $x + 2y + 3z = 0$ $y + 2z = 0$ $-3y - 6z = 0$ Replace the third equation by itself plus three times the second: $x + 2y + 3z = 0$ $y + 2z = 0$ $0 = 0$ This last statement is clearly true. The system is thus, dependent. There are infinitely many solutions: Let z be any real number. $y = -2z$ $x = -2y - 3z =$ $-2(-2z) - 3z = z$ Thus the solutions are $x = z$, $y = -2z$, $z = z$ for any real number z .

Problem	Hint	Solution
12. Let $p(x) = x^4 + x^3 + 10x^2 + 9x + 9$ Given that $3i$ is one root of $p(x)$, find the other roots and factor $p(x)$.	If $3i$ is one root then its conjugate is another root.	Since complex roots come in conjugate pairs, two roots are $3i$ and $-3i$. A factor of $p(x)$ is then $(x-3i)(x+3i) = x^2 + 9$. To find the other factors, we use long division to divide $x^2 + 9$ into $p(x)$: $$\begin{array}{r} x^2 + x + 1 \\ x^2+9 \overline{)\, x^4+x^3+10x^2+9x+9} \\ \underline{x^4 \quad\quad + 9x^2} \\ x^3 + x^2 + 9x + 9 \\ \underline{x^3 \quad\quad + 9x} \\ x^2 \quad + 9 \\ \underline{x^2 \quad + 9} \\ 0 \end{array}$$ Thus $p(x) = (x^2+9)(x^2+x+1)$ Now, $x^2 + x + 1 = 0$ when $x = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{-3}}{2}$ $= \frac{-1 \pm \sqrt{3}\, i}{2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$ Thus the roots of $p(x)$ are $3i$, $-3i$, $-\frac{1}{2} + \frac{\sqrt{3}}{2} i$, $-\frac{1}{2} - \frac{\sqrt{3}}{2} i$
13. Compute the discriminant of the quadratic equation $x^2 + 4x - 2 = 0$ and discuss the nature of the roots.	Discriminant: $b^2 - 4ac$	Since $a = 1$, $b = 4$, $c = -2$ $b^2 - 4ac = 4^2 - 4(1)(-2) =$ $= 24$ Since 24 is positive, the roots are real and distinct. Since 24 is not a perfect square, the roots are irrational.

Problem	Hint	Solution
14. Given that $\sin\theta = \frac{4}{5}$, and $0 < \theta < \frac{\pi}{2}$. Find: a) $\sin(\theta + 45^\circ)$ b) $\cos(\theta - \frac{\pi}{3})$ c) $\tan(\theta + \frac{\pi}{6})$	$\sin(A + B) = ?$ $\cos(A - B) = ?$ $\tan(A + B) = ?$ 	a) $\sin(\theta + 45^\circ) =$ $\sin\theta\cos 45^\circ + \cos\theta\sin 45^\circ$ $= \frac{4}{5}\frac{\sqrt{2}}{2} + \frac{3}{5}\frac{\sqrt{2}}{2} = \frac{7}{5}\frac{\sqrt{2}}{2} = \frac{7\sqrt{2}}{10}$ b) $\cos(\theta - \frac{\pi}{3}) =$ $\cos\theta\cos\frac{\pi}{3} + \sin\theta\sin\frac{\pi}{3}$ $= \frac{3}{5}\frac{1}{2} + \frac{4}{5}\frac{\sqrt{3}}{2} = \frac{3}{10} + \frac{4\sqrt{3}}{10}$ c) $\tan(\theta + \frac{\pi}{6}) =$ $\frac{\tan\theta\tan\frac{\pi}{6}}{1 - \tan\theta\tan\frac{\pi}{6}} =$ $\frac{\frac{4}{3}\frac{\sqrt{3}}{3}}{1 - \frac{4}{3}\frac{\sqrt{3}}{3}} = \frac{\frac{4\sqrt{3}}{9}}{\frac{9 - 4\sqrt{3}}{9}} =$ $\frac{4\sqrt{3}}{9 - 4\sqrt{3}}$
15. Graph the function $f(x) = 2^x$. Is it even, odd, or neither?	Fill in the table 	 $f(-x) = 2^{-x} = 1/(2^x)$ This does not equal $f(x)$ or $-f(x)$, so the function $f(x)$ is neither even nor odd.

Problem	Hint	Solution
16. Solve: $\frac{4 - 2x}{(x + 6)(x - 3)} \geq 0$	Mark off on the number line the zeros of the numerator and of the denominator. Test points in each interval.	 Thus, the left side is greater than or equal to zero when $x < -6$ or $2 \leq x < 3$.
17. One day a store sold 60 blouses. Blue ones were sold for \$10.50 each and white ones for \$9.75 each. In all \$615 worth of blouses were sold. How many blouses of each color were sold?	Let x = number of blue blouses sold. Let y = number of white blouses sold. Find two equations in x and y.	$x + y = 60$ $10.50x + 9.75y = 615$ From the first equation we see that $y = 60 - x$ Substituting this into the second equation yields: $10.50x + 9.75(60 - x) = 615$ $10.50x + 585 - 9.75x = 615$ $.75x = 30$ $x = \frac{30}{.75} = \frac{3000}{75} = 40$ $y = 60 - 40 = 20$
18. Solve $\log_a x^3 = 3$	Write it in exponential form.	$a^3 = x^3$ $a = x$
19. Assume that x and y are real numbers. Solve for x and y $x + 3y + 2i = 6 + (2x + y)i$	$a + bi = c + di$ if and only if $a = c$ and $b = d$.	$x + 3y = 6$ $2x + y = 2$ Multiplying the first equation by -2: $-2x - 6y = -12$ $2x + y = 2$, adding, $-5y = -10$, $y = 2$, $x + 3(2) = 6$, $x = 0$

Problem	Hint	Solution
20. Evaluate each of the following: a) $\csc\left(\frac{-\pi}{2}\right)$ b) $\sin(-\pi)$ c) $\cos 360^\circ$ d) $\csc\frac{3\pi}{2}$		a) $-\frac{\pi}{2}$ intersects the unit circle at point D, so $\csc\left(-\frac{\pi}{2}\right) = \frac{1}{\sin\left(-\frac{\pi}{2}\right)} = \frac{1}{-1} = -1$ b) $-\pi$ intersects the unit circle at point C. Thus, $\sin(-\pi) = 0$. c) 360° intersects the unit circle at point A. Thus, $\cos 360^\circ = 1$. d) $\frac{3\pi}{2}$ intersects the unit circle at point D. Thus, $\csc\frac{3\pi}{2} = \frac{1}{\sin\frac{3\pi}{2}} = \frac{1}{-1} = -1$
21. Solve: $\lvert 1 - 9x \rvert < -6$	Is this possible?	By definition, an absolute value can never be negative. There are no solutions to this inequality.
22. Solve: $x^2 - 3x - 10 > 0$	Sketch the graph of the function $y = x^2 - 3x - 10$ $= (x-5)(x+2)$ and see where $y > 0$, (where the graph lies above the x axis).	 We see that $y > 0$ when $x < -2$ or $x > 5$.
23. Express the following in terms of logarithms of x, y, and z: $\log_a \sqrt[9]{\frac{xy}{z^3}}$	$\log_a(uv) = ?$ $\log_a(u/v) = ?$ $\log_a(u^r) = ?$	$\log_a\left[\left(\frac{xy}{z^3}\right)\right]^{\frac{1}{9}} = \frac{1}{9}\left[\log_a\left(\frac{xy}{z^3}\right)\right] =$ $\frac{1}{9}\left[\log_a(xy) - \log_a(z^3)\right] =$ $\frac{1}{9}\left[\log_a x + \log_a y - 3\log_a z\right]$

Problem	Hint	Solution
24. Evaluate each of the following: a) $\sin 405^\circ$ b) $\tan \frac{17\pi}{6}$ c) $\csc -\frac{8\pi}{3}$ d) $\cot 45^\circ$		a) 405° intersects the unit circle at point A . Thus, $\sin 405^\circ = \frac{\sqrt{2}}{2}$. b) $\frac{17\pi}{6}$ intersects the unit circle at point B . Thus, $\tan \frac{17\pi}{6} = \frac{\sin \frac{17\pi}{6}}{\cos \frac{17\pi}{6}} =$ $\frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{-1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$ c) $-\frac{8\pi}{3}$ intersects the unit circle at point C . Thus, $\csc(-\frac{8\pi}{3}) = 1/(\sin(-\frac{8\pi}{3}) =$ $1/(-\frac{\sqrt{3}}{2}) = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$ d) 45° intersects the unit circle at point A . So, $\cot 45^\circ = \frac{\cos 45^\circ}{\sin 45^\circ} =$ $(\sqrt{2}/2)/(\sqrt{2}/2) = 1$
25. Let $y = -\cos(\frac{x}{3} + \pi)$. Find the amplitude period and phase shift. Sketch one cycle of the graph.	$y = -\cos \frac{1}{3}(x + 3\pi)$	period: $\frac{2\pi}{\frac{1}{3}} = 6\pi$ amplitude: $\lvert -1 \rvert = 1$ phase shift: -3π (to left) 

Problem	Hint	Solution
26. Express in the form a + bi a) $\frac{\sqrt{5}+3i}{\sqrt{5}-3i}$ b) $i^{40}-i^{51}-i^{122}+i^{53}$	a) Multiply the numerator and the denominator of the fraction by the complex conjugate of the denominator. b) To find i^k, divide k by 4. Let r be the remainder. If r than i^k 0 1 1 i 2 -1 3 -i	a) $\frac{\sqrt{5}+3i}{\sqrt{5}-3i}\cdot\frac{\sqrt{5}+3i}{\sqrt{5}+3i} = \frac{5+3\sqrt{5}i+3\sqrt{5}i+9i^2}{5-3\sqrt{5}i+3\sqrt{5}i-9i^2} = \frac{(5-9)+6\sqrt{5}i}{5+9} = \frac{-4+6\sqrt{5}i}{14} = \frac{-2+3\sqrt{5}i}{7}$ b) k r i^k 40 0 1 51 3 -i 122 2 -1 53 1 i Thus $i^{40}-i^{51}-i^{122}+i^{53} =$ $1-(-i)-(-1)+i =$ $2+2i$
27. Solve: $\log_2 x + \log_2(x+2) = 3$	Simplify using the rules of logarithms.	$\log_2 x + \log_2(x+2) =$ $\log_2(x(x+2)) = \log_2(x^2+2x) = 3$ In exponential form: $x^2+2x = 2^3 = 8$ $x^2+2x-8=0$ $(x+4)(x-2)=0$ $x=-4$ or $x=2$. But $\log_2(-4)$ is undefined $x=-4$ is rejected. The solution is $x=2$.

Problem	Hint	Solution
28. A subcompact car uses a tire whose radius is 14 in. a) How far has the car moved when the tire completes one rotation? b) How many rotations are completed when the tire has traveled one mile?	$s = \theta r$ where θ is measured in radians	one rotation = $360^\circ = 2\pi$ Since $s = \theta r$ $s = 2\pi \cdot 14 = 28\pi = 87.9$ in b) $s = \theta r$ 1 mile = 63360 in $63360 = \theta \times 14$ $\theta = \frac{63360}{14} = 4525.7$ rdn $= \frac{4525.7}{2\pi} = 720.3$ rotations
29. Find all the roots, with multiplicities of $p(x) =$ $(x^2-6x+9)(x^2-25)$	Set each quadratic factor equal to zero.	$x^2 - 6x + 9 = 0$ $(x - 3)^2 = 0$ $x = 3$ (double root) $x^2 - 25 = 0$ $x^2 = 25$ $x = -5, x = 5$ The roots are: 3 with a multiplicity of 2 5 and -5.